Smith-Purcell Radiation with Different Grating Parameters and Beam Bunching Frequencies

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Abstract: *This paper calculates Smith-Purcell radiation (SPR) operating points based on dispersion curves at different grating parameters (groove's heights and widths) with a fixed grating period. The operating point is determined at the intersection of the SPR dispersion and the beam mode, which emits SPR radiation at second harmonics of this frequency. According to the SPR equation, the SPR frequency depends on the period of the metallic grating. The dispersion relation, on the other hand, shows that, in addition to the grating period, the frequency of SPR varies significantly with grating parameters. Numerical analysis using CST Particle Studio shows that when the grating is excited with a beam having a bunching frequency close to the SPR frequency calculated from the dispersion relation, the radiation field has significantly higher strength than excited with beams of other bunching frequencies.*

Keywords: Smith-Purcell radiation, grating optimization, beam circuit interaction, beam bunching

Introduction

High-power, efficient, and low-cost electromagnetic sources have significant uses in high-resolution imaging, biomedical scanning, material analysis, security systems, and high-rate data communications, etc. Back wave oscillation (BWO) based Smith-Purcell Radiation (SPR) has attracted strong interests for producing Terahertz (THz) radiation. When an electron beam emitted with the velocity v above a metallic periodic grating of period L, SPR can be generated with wavelength λ at an angle θ. (Fig. 1), described by [1],

$$\lambda = \frac{L}{n}\left(\frac{1}{\beta} - \cos\theta\right). \quad (1)$$

Here, n is an integer, $\beta (= v/c)$ is Lorentz factor of the electron beam with velocity v, and c is the speed of light.

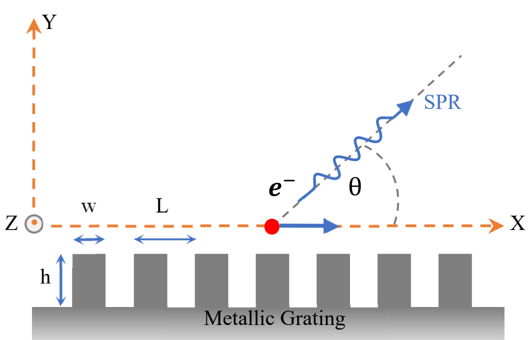

Figure 1. Schematic of SPR grating and beam configuration.

In this paper, we present the Smith-Purcell radiation (SPR) operating points for different grating parameters (groove heights and widths) with a fixed grating period based on the dispersion relation (Equation 2) [2]. The effects of different beam bunching frequencies on SPR radiation were studied using the PIC solver of CST Particle Studio.

Dispersion Relation

The dispersion equation for the SPR model shown in Fig. 1 is [2,3],

$$\frac{\cot(\bar{\omega}\bar{H})}{\bar{\omega}\bar{H}} - \sum_{n=-\infty}^{\infty}\left(\frac{\sin\theta_n}{\theta_n}\right)^2 \frac{\bar{W}}{\gamma_n \bar{H}} = 0 \quad (2)$$

where the normalized grating width $\bar{W} = w/L$, grating height $\bar{H} = h/L$, frequency $\bar{\omega} = \omega L/c$, wavenumber $\bar{k} = kL$, $\theta_n = P_n\bar{W}/2$, $\gamma_n = \sqrt{P_n{}^2 - \bar{\omega}^2}$, and $P_n = \bar{k} + 2n\pi$. Equation 2 is for the evanescent (surface mode) wave that is derived by assuming TEM mode inside the grooves and Floquet fields outside the grating [3].

TABLE I. Main parameters for numerical analysis.

Beam energy	50 eV
Beam Height from Grating Surface	10 μm
Beam thickness	20 μm
Beam Current	0.01 A
Grating period, L	120 μm
Grating groove height, h	100 μm
Grating groove width, w	60 μm
Structure width (in Z direction)	100 μm
Number of grating periods	35

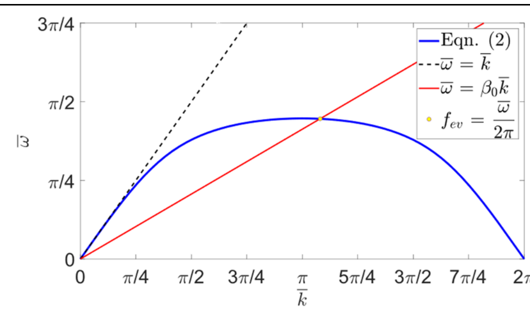

Figure 2. The dispersion relation $\omega(k)$ (Eqn. 2) of the grating structure, light line $(\bar{\omega} = \bar{k})$, beam line $(\bar{\omega} = \beta_0\bar{k})$ and operating point for SPR (f_{ev}) are shown using the parameters in the Table I.

The operating frequency of the BWO oscillation (evanescent surface mode) frequency (f_{ev}) for the grating parameters stated in Table 1 is calculated analytically by the intersection of the beam line $(\bar{\omega} = \beta_0\bar{k})$ to the dispersion relation (Equation 2), as shown in Fig. 2 [3], for the beam velocity to light speed radio $\beta_0 = 0.4126$. At the oscillating point, the slope $(d\bar{\omega}/d\bar{k})$ or the group velocity of the dispersion curve is negative, hence the backward wave will interact with the beam [2]. As a result, SPR radiation occur at the corresponding second harmonic of this frequency.

The Effect of Grating Parameters

Figure 3 shows the SPR dispersion curves at different grating groove's heights, with the width fixed at 60 μm and all the other parameters remaining the same as in Table I. Similarly, in Fig. 4 we show SPR dispersion curves at different value of the groove's width, with the height fixed at 100 μm and all the other parameters remaining as in Table I.

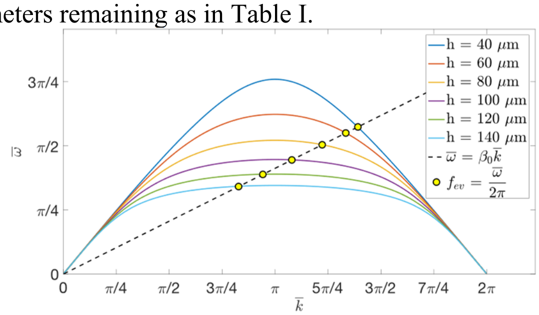

Figure 3. The dispersion relation $\omega(k)$ (Eqn. 2) for different groove's height *h*. Here groove's width *w* is fixed at 60 μm and other parameters are kept as the same as in Table I.

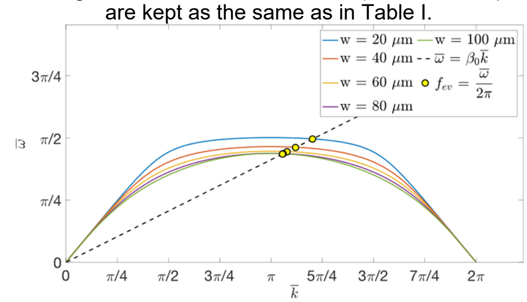

Figure 4. The dispersion relation $\omega(k)$ (Eqn. 2) for different groove's width *w*. Here groove's height *h* is fixed at 100 μm and other parameters are kept as the same as in Table I.

From Figs. 3 and 4, it is clear that with fixed beam energy and grating period, the operation point at the evanescent wave frequency f_{ev} varies significantly with the change of grating groove's width and height. The corresponding evanescent wavelength λ_{ev} is shown in Fig. 5 [2] and Fig. 6 shows the operating frequency (f_{ev}) and evanescent wavelength (λ_{ev}) for a wide range of grating groove's height and width.

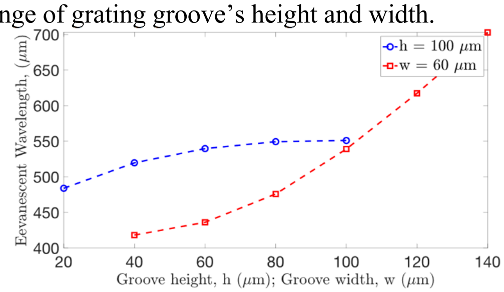

Figure 5. The corresponding evanescent wavelength (λ_{ev}) as a function of grating groove's height and width, for Figures 3 [red das line] and 4 [blue das line] [2].

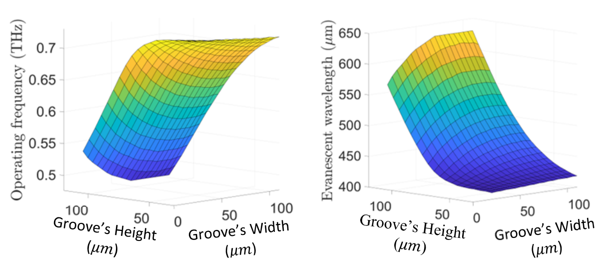

Figure 6. The surface plot of the corresponding operating frequency (f_{ev}) and evanescent wavelength (λ_{ev}) for a wide range of grating groove's height and width.

The Effect of Beam Bunching

Coherent SPR frequency for the parameters in Table I was found to be $2f_{ev}$ = 1.09 THz when excited by a continuous beam with sufficiently large current [2]. Here, we use CST Particle Studio's PIC solver to study the effect of beam bunching frequency on the radiation frequency. We excite the grating using more than 100 bunches of rectangular shaped pre-bunched electron beam of 0.01 A, with varying bunching period (T_b) from 600 ps to 1200 ps with an incremental step of roughly 25 ps, where the bunch length is kept at $T_b/2$.

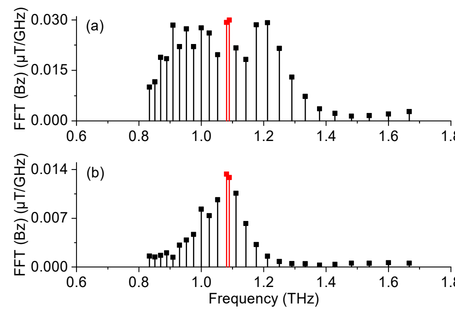

Figure 7. FFT of detected magnetic field B_z (a) at 3 mm above and (b) at 7 mm above from the midpoint of the grating in Table I over the beam bunching period (T_b) from 600 ps to 1200 ps for more than 100 bunches, where the length of each bunch is fixed at $T_b/2$ with bunch current of 0.01A.

Figures 7(a) and 7(b) show the Fast Fourier Transform (FFT) of the detected magnetic field B_z at 3 mm and 7 mm above the center of the grating, respectively. The results show that the radiation frequency is largely determined by the corresponding electron bunching frequency. At 3 mm above the grating the magnitude of the radiation field is insensitive to the radiation frequency (or bunching frequency). However, at 7 mm above the grating, significantly higher radiation field is detected at 1.081 THz (red) and 1.0904 THz (red), whose frequencies closely align with the SPR frequency determined from the dispersion relation (Eqn. 2) of the model in Table I.

Conclusion

In summary, we show how the operating frequencies of SPR have changed with different grating parameters using dispersion relations where the period has kept fixed. The CST simulation results show significantly higher radiation strength when the beam bunching frequency aligns the SPR frequency determined from the dispersion relation. The results provide insight into optimal designing of SPR radiation.

Acknowledgment

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References

1. S. J. Smith and E. M. Purcell, "Visible Light from Localized Surface Charges Moving across a Grating," Physical Review, vol. 92, no. 4, pp. 1069-1069, 1953.
2. P. Zhang, L. K. Ang, and A. Gover, "Enhancement of coherent Smith-Purcell radiation at terahertz frequency by optimized grating, prebunched beams, and open cavity," Physical Review Special Topics - Accelerators and Beams, vol. 18, no. 2, p. 020702, 2015.
3. P. Zhang, B. Hoff, Y. Y. Lau, D. M. French, and J. W. Luginsland, "Excitation of a slow wave structure," Phys. Plasmas, vol. 19, no. 12, p. 123104, 2012.