Application of Electrodynamic Admittances in the TWT Theory

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Introduction

Unlike the conventional transmission lines and waveguides, in which, as a rule, the wave propagates only in one region, in slow-wave structures (SWSs) with the electron beam (e-beam) the wave propagates or concentrates at least in three regions. Besides, in the most cases slow waves are represented by components of the E- and H-waves, each of them with different boundary conditions. This doubles the number of the boundary conditions, which achieves 16 for two regions SWSs plus 4 boundary conditions for the e-beam.

Physics of Electrodynamic Admittances

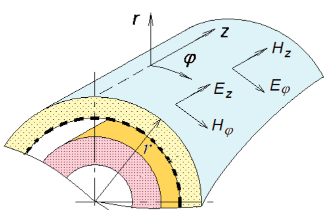

Figure 1. Projections of the E- and H-waves' components on the cylindrical surface.

Each electromagnetic wave, propagating along a transmission line, can be characterized by the *electrodynamic impedance* $\tilde{Z}$ determined as $\tilde{Z} = \vec{E}/\vec{H}$, where $\vec{E}$ and $\vec{H}$ are vectors of the electric and magnetic intensities and $\tilde{Z}$ represents the second order symmetric tensor. Although the scalar value of the wave impedance is widely used in the circuit theory, in the case of its electrodynamic version, it is much more convenient to use its inverse value, the *electrodynamic admittance* $\tilde{Y}$ defined as $\tilde{Y} = \vec{H}/\vec{E}$. In general, the projection of this tensor on the boundary surface of axially symmetric SWS (Fig. 1) is the two-dimensional tensor with two vectors and four scalar projections [1]. If surface conductivities coincide with directions of coordinates, the projection on a boundary surface or a parallel to it surface are scalar values, $Y^e(r)$ and $Y^m(r)$, determined as

$$Y^e(r) = -H_\varphi(r)/E_z(r),\ \ Y^m(r) = H_z(r)/E_\varphi(r). \tag{1}$$

Here the upper script *e* designates the electric type admittance, whereas *m* designates the magnetic type admittance. Note that whereas the direction of the vector product of the H-wave components coincides with the direction of coordinate *r*, the direction of the E-wave components' vector product is opposite. This explains the minus in the expression for $Y^e(r)$.

Boundary Conditions

As follows from the boundary conditions, in the absence of surface currents at the boundary surface, the tangential components of the electric and magnetic fields' intensities are continuous. This gives

$$Y^e_{n-1}(a) = Y^e_n(a),\ \ Y^m_{n-1}(a) = Y^m_n(a), \tag{2}$$

where *n*-1 and *n* are numbers of the neighbor regions with the boundary at $r = a$ (Fig. 2). Note that each of equalities (2) replaces two boundary conditions for tangential components of the E- and H-waves decreasing twice the boundary conditions at the each boundary.

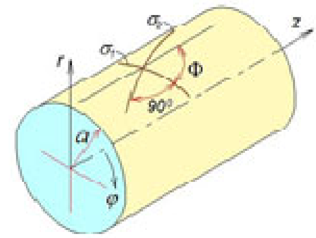

Figure 2. Sheath helix with finite values of conductivities.

In the presence of the surface currents, tangential components change at the value of the surface current's density and we have

$$\Delta Y^e(a) = -i_z/E_z(a),\ \ \Delta Y^m(a) = -i_\varphi/E_\varphi(a), \tag{3}$$

where Δ designates jumps of the admittances in the positive direction of r, whereas i_z, i_φ are components of the surface currents' densities. If directions of the surface conductivities coincide with directions of coordinates *z* and φ,

$$\Delta Y^e(a) = -\sigma_z,\ \ \Delta Y^m(a) = -\sigma_\varphi. \tag{4}$$

Here σ_z and σ_φ are components of the surface conductivity tensor Σ.

Equation of the Sheath Helix with the Finite Conductivities

If, as it takes place in the case of the helix, directions of conductivities do not coincide with directions of coordinates and the surface conductivity σ_1 at the angle Φ to the longitudinal axis as well as the surface conductivity σ_2 in the perpendicular direction (Fig. 2) are finite [1],

$$i_z = \sigma_{z\varphi}E_\varphi(a) + \sigma_{zz}E_z(a),\ \ i_\varphi = \sigma_{\varphi\varphi}E_\varphi(a) + \sigma_{\varphi z}E_z(a), \tag{5}$$

where the scalar projections of symmetric tensor Σ are determined by the next expressions:

$$\sigma_{\varphi\varphi} = \sigma_1 \sin^2\Phi + \sigma_2\cos^2\Phi,\ \ \sigma_{zz} = \sigma_1\cos^2\Phi + \sigma_2\sin^2\Phi,$$

$$\sigma_{\varphi z} = \sigma_{z\varphi} = (\sigma_1 - \sigma_2)\sin\Phi\cdot\cos\Phi. \tag{6}$$

Substituting (5) in relations (3), we obtain

$$\Delta Y^e(a) + \sigma_{zz} = -\sigma_{z\varphi}E_\varphi(a)/E_z(a),\ \ \Delta Y^m(a) + \sigma_{\varphi\varphi} = -\sigma_{\varphi z}E_z(a)/E_\varphi(a). \tag{7}$$

Excluding ratio $E_z(a)/E_\varphi(a)$ we receive with the help of (5) - (7) the general dispersion equation of the helix with losses

$$\frac{\sigma_{\varphi\varphi}}{\sigma_{zz}}\frac{\Delta Y^e(a)}{\Delta Y^m(a)} + \frac{\sigma_1\sigma_2}{\sigma_{zz}\Delta Y^m(a)} + \frac{\Delta Y^e(a)}{\sigma_{zz}} + 1 = 0. \tag{8}$$

In the most cases, conductivity σ_2 is relatively small or equals to zero and $\sigma_{\varphi\varphi}/\sigma_{zz} \approx \tan^2\Phi$. This gives

$$\frac{\Delta Y^e(a)}{\Delta Y^m(a)}\tan^2\Phi + \frac{\Delta Y^e(a)}{\sigma_1\cos^2\Phi} + 1 \approx 0. \tag{9}$$

References

1. Loshakov, L. N. and Yu. N. Pchelnikov, *Theory and the gain calculation of the traveling-wave tube*, Moscow, Russia: Soviet Radio, 1964, (in Russian).