Dispersion Relation of Embed Beam-Wave Interaction for Planar Grating Structure Terahertz Radiation Source

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Abstract: *In vacuum electronic devices region, high power, compact, portable and miniaturized terahertz radiation source has always been the research goal. The core of vacuum electronic device is slow wave structure, which determines the beam wave interaction efficiency and output power. For the classical slow wave structure of rectangular grating, the field matching method is used to analyze the slow wave structure of rectangular grating with holes in the middle, and then the dispersion equation is obtained. For the field treatment in the trough, the higher term is retained, which is expressed as the sum of an infinite standing wave [1].*

Keywords: Terahertz radiation source; rectangular grating; electron beam; slow wave system; backward wave tube; dispersion characteristics

Introduction

Among many slow wave structures, rectangular gate slow wave structure is widely used in microwave and millimeter wave devices because of its compact structure, simple fabrication process and high processing accuracy, which can work in sub millimeter wave band or even terahertz wave band [2]. Such as backward wave oscillator, traveling wave tube, klystron [3]. One of its most attractive advantages is the measurability of the gate structure in the case of small size and short wavelength [4]. Because most of the area of the electron beam is located in the strong field, the output power of the electron beam embedded grating is higher than that of the ordinary plane rectangular grating. Therefore, this paper uses the field matching method to obtain the high frequency characteristics of the rectangular grating slow wave system.

Dispersion Relation without Electron Beam

The rectangular gate structure is shown in Figure 1. The height of the rectangular waveguide is b, the width is w, the gate period is d, the slot depth is p, and the slot width is s. In a single cycle, there are three zones: Zone 1 is the transmission zone($0 < x < b, 0 < z < d$), zone 2 is the slot zone($-p < x < 0, 0 < z < s$), zone 3 is the opening zone($-q < x < 0, -\frac{u}{2} < y < \frac{u}{2}, s < z < d$).

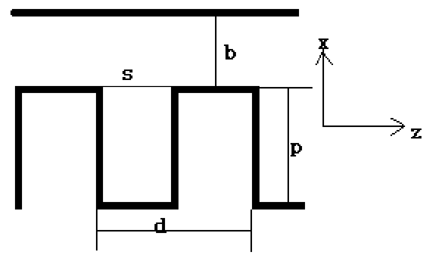
In zone 1, because the waveguide is periodic in the z-axis direction, according to Floquet's theorem and the boundary conditions, the electric and magnetic components can be written as [5]

$$H_y^1(x,y,z) = \sum_{n=-\infty}^{+\infty} B_n^1 \cosh[v_n(b-x)]\cos(k_y y)e^{-jk_n z} \tag{1}$$

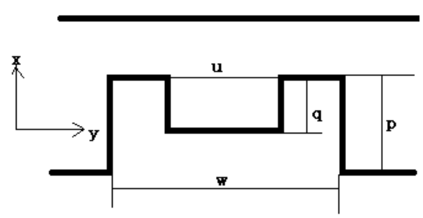
$$E_z^1(x,y,z) = \sum_{n=-\infty}^{+\infty} \frac{j\omega\mu_0 v_n}{k_0^2 - k_y^2} B_n^1 \sinh[v_n(b-x)]\cos(k_y y)e^{-jk_n z} \tag{2}$$

k_n and v_n given by

$$k_n = k_z^1 + \frac{2\pi n}{d}, (v_n)^2 = (k_n)^2 - k_0^2 + k_y^2$$



(a) Longitudinal section



(b) Cross section

Fig. 1 Structure of rectangular grating

Using the same method, we can get the electric and magnetic field components in region 2 and region 3.

All the troughs are approximated as standing wave fields. At x = 0, from the tangent continuity condition of electric field

$$E_z^1(0,y,z) = \begin{cases} E_z^2(0,y,z), & 0 \le z \le s, -\frac{w}{2} < y < \frac{w}{2} \\ 0, s < z < d, -\frac{w}{2} < y < -\frac{u}{2}, \frac{u}{2} < y < \frac{w}{2} \\ E_z^3(0,y,z), & s < z < d, -\frac{u}{2} < y < \frac{u}{2} \end{cases} \tag{3}$$

From the continuity of magnetic field

$$H_y^1(0,y,z) = H_y^2(0,y,z), 0 < z < s \tag{4}$$

$$H_y^1(0,y,z) = H_y^3(0,y,z), s < z < d, -\frac{u}{2} < y < \frac{u}{2} \tag{5}$$

Bring the relevant magnetic field components into equation (3) (4) and (5), and then solve the equations by eigenfunction method, we can get the "cold" dispersion equation:

$$\frac{v_n}{k_0^2-k_y^2}B_n^1\sinh(v_n b)\sin(\frac{l\pi}{2})d$$
$$=-\sum_{m=0}^{+\infty}\frac{v_m}{k_0^2-k_y^2}\sum_{n'=-\infty}^{+\infty}B_{n'}^1\cosh(v_{n'}b)\tanh(v_m p)\sin(\frac{l\pi}{2})\frac{R(-k_{n'},k_m,s)R(k_n,k_m,s)}{(1+\delta_{m0})}\frac{2}{s}$$
$$-\sum_{t=-\infty}^{+\infty}\frac{v_t^3}{k_0^2-(\frac{l\pi}{u})^2}B_t^1\frac{\cosh(v_t b)}{\cosh(v_t^3 q)}\sinh(v_t^3 q)\sin(\frac{l\pi u}{2w})A(k_n,k_t) \tag{6}$$

When the width u of the opening area approaches 0 or the depth q of the opening area approaches 0, we can get the "cold" dispersion equation of planar metal grating without holes, and the result is consistent with reference [6].

$$B_n^1 v_n\sinh(v_n b)d=-\sum_{n'=-\infty}^{+\infty}B_{n'}^1\cosh(v_{n'}b)R(-k_{n'},k_m,s)\sum_{m=0}^{+\infty}v_m\tanh(v_m p)\frac{2R(k_n,k_m,s)}{s(1+\delta_{m0})} \tag{7}$$

Dispersion Relation of Loaded Electron Beam

The slow wave structure of rectangular gate waveguide with electron beam is shown in Figure 2. The height of rectangular waveguide is b, the width is w, the period of metal gate is d, the depth of slot is p, and the width of slot is s. At x = - c, a rectangular electron beam with a thickness of a + c, a width of L, and a current density of J is introduced. The electron beam assumes that the electron moves only longitudinally (in the z-axis direction), but not transversely.

For the convenience of calculation, the groove area is approximated as a standing wave field, and the rectangular grating is divided into five regions.

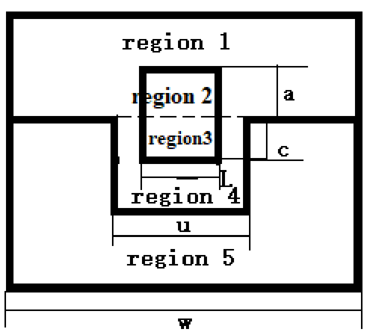


Figure 2 Schematic diagram of rectangular grating waveguide structure when electron beam is loaded

Similar to the method of deriving the "cold" dispersion equation, the field expressions for each region are written according to the boundary conditions. The fields in regions 2 and 3 belong to the electron beam region. Due to the influence of the electron beam, the electric field component needs to be reconstructed.

$$E_z^2(x,y,z)=-\sum_{n=-\infty}^{+\infty}\frac{j\omega\mu_0 v_{an}}{(\chi_n^w k_0^2-k_y^2)}[A_n^2\cosh(v_{an}x)+B_n^2\sinh(v_{an}x)]\cos(k_y y)e^{-jk_n z} \tag{8}$$

$$E_z^3(x,y,z)=-\sum_{n=-\infty}^{+\infty}\frac{j\omega\mu_0 v_{cn}}{[\chi_n^u k_0^2-(\frac{l\pi}{u})^2]}[A_n^3\cosh(v_{cn}x)+B_n^3\sinh(v_{cn}x)]\cos(\frac{l\pi}{u}y)e^{-jk_n z} \tag{9}$$

In the above equations,Where $\omega_{pe},\chi_n^u,\chi_n^w,J_{Lw}$ and J_{Lu} are parameters related to the electron beam.

$$(v_{an})^2=\frac{(k_0^2\varepsilon_{en}-k_y^2)}{k_0^2-k_y^2}(v_n)^2,(v_{cn})^2=\frac{(k_0^2\varepsilon_{en}-k_y^2)}{k_0^2-(\frac{l\pi}{u})^2}(v_n)^2$$

After obtaining the field components of each region, combining the boundary conditions and continuity conditions at x = a, x = 0, x = -c, the final dispersion equation can be obtained as

$$B_n^1\{\frac{v_n}{k_0^2-k_y^2}\sinh(v_n b)[\sin(\frac{l\pi}{2})-\sin(\frac{l\pi L}{2w})]-\frac{v_{an}}{(\chi_n^w k_0^2-k_y^2)}N_n^1\sin(\frac{l\pi L}{2w})\}d$$
$$=-\sum_{m=0}^{+\infty}\frac{v_m}{k_0^2-k_y^2}\sum_{n'=-\infty}^{+\infty}B_{n'}^1\{\cosh(v_{n'}b)[\sin(\frac{l\pi}{2})-\sin(\frac{l\pi L}{2w})]+M_{n'}^1\sin\frac{l\pi L}{2w}\}$$
$$\frac{2R(-k_{n'},k_m,s)}{s(1+\delta_{m0})}\tanh(v_m p)R(k_n,k_m,s)-\sum_{t=-\infty}^{+\infty}B_t^1\frac{C_t^1}{C_t^4}\{\frac{v_t^4}{k_0^2-(\frac{l\pi}{u})^2}\sinh(v_t^4 q)$$
$$[\sin(\frac{l\pi}{2})-\sin(\frac{l\pi L}{2u})]+\frac{v_{ct}}{[\chi_t^u k_0^2-(\frac{l\pi}{u})^2]}N_t^4\sin(\frac{l\pi L}{2u})\}A(k_n,k_t) \tag{10}$$

In the equation (10), $M_n^1,N_n^1,N_n^4,\mathrm{M}_n^4,C_n^1$ and C_n^4 are parameters about the angular frequency and the longitudinal phase-shift constant.

When the thicknesses a and c of the electron beams approach 0, the dispersion equation is consistent with the "cold" dispersion equation (7), which proves that the "hot" dispersion equation is correct.

Conclusion

In this paper, the eigenfunction method is used to obtain the "cold" dispersion and "hot" dispersion equations of the electron beam embedded in the rectangular grating by field matching method, and the correctness of the dispersion equations is verified. The derivation of the dispersion equation of the electron beam embedded rectangular grating provides a basis for future research on the high frequency characteristics and beam-wave interaction characteristics of the rectangular grating, and it has guiding significance for the design of the BWO which has the electron beam embedded grating structure.

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