Kinetic Analysis of Two Dimensional Cyclotron Maser with Single Gratings

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Abstract: *In this paper, the TE mode dispersion relation of the two dimensional sheet beam cyclotron maser with single gratings is obtained by the kinetic method. By numerical calculations, the effects of the structure parameters and the beam state on the growth rate are analyzed. As the high-order mode in the two dimensional single grating waveguide, the TE mode has the value to be investigated and the method is meaningful in the practice.*

Keywords: rectangular grating; linear theory; cyclotron maser.

Introduction

Due to the urgent demands of modern radar and electronic countermeasure systems for high-power, wide-band, high-frequency vacuum devices, the sheet beam rectangular grating waveguide system has become a hot topic at home and abroad. The physical mechanism of the sheet beam rectangular grating waveguide system usually is the Cherenkov resonance [1-2]. In order to further improve the gain and the bandwidth of the system, a cyclotron sheet beam can be used in the system [3]. Moreover, compared with the fundamental mode, operating in the high-order modes can obtain larger size, higher power capacity and stronger heat dissipation at the same operating frequency.

In this paper, the kinetic analysis of the two dimensional cyclotron maser with single gratings is presented. In the two dimensional single grating waveguide, the TE mode is the high-order mode. As a result, it is meaningful to investigate the TE mode two dimensional cyclotron maser with single gratings.

Model and Formulae

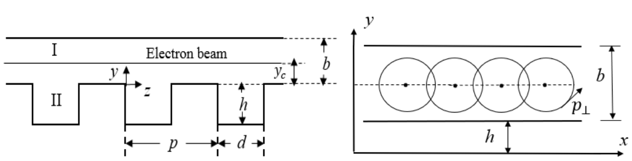

Figure 1. The cross section of the two dimensional metallic grating waveguide with a sheet electron beam.

Figure 1 shows the cross section of the 2-D grating waveguide with a cyclotron sheet electron beam, which is infinite in the x direction. b is the height of the waveguide with the grating period p, groove width d and groove depth h on the bottom wall. Space I is the region of empty waveguide and Space II is the region of gratings.

In the following analysis, an exp(-jωt) time dependence is assumed for all field quantities. The field components of the TE mode in Space I can be expressed as follows.

$$E_x^I = \sum_n a_n \frac{1}{\sinh(v_n b)} \sinh[v_n(b-y)] e^{jk_n z} \tag{1a}$$

$$H_y^I = \frac{1}{\omega\mu} \sum_n a_n k_n \frac{1}{\sinh(v_n b)} \sinh[v_n(b-y)] e^{jk_n z} \tag{1b}$$

$$H_z^I = -j\frac{1}{\omega\mu} \sum_n a_n v_n \frac{1}{\sinh(v_n b)} \cosh[v_n(b-y)] e^{jk_n z} \tag{1c}$$

where a_n is the field amplitude of the n-th spatial harmonic,

$$k_n = k_z + 2n\pi / p \quad (n = 0, \pm 1, \pm 2, \ldots) \tag{2}$$

$$v_n^2 = k_n^2 - k_0^2 \tag{3}$$

k_z is the wave number in z direction.

According to the linear Vlasov equation, the electron distribution f_1 can be obtained. Then, the perturbed beam J_x can be given by f_1 through $\vec{J} = -e\int_{\vec{p}} \vec{v} f_1 d^3 p$. After substituting the perturbed beam into the Maxwell equation and integrating by parts, the hot dispersion relation of the initial distribution function $f_0 = \frac{N_0}{2\pi p_{\perp 0}} \delta\left(p_\perp - p_{\perp 0}\right) \delta\left(p_z - p_{z0}\right) \delta\left(y_c - y_0\right)$ can be obtained

$$\begin{aligned} D(\omega,\beta) = & \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} \frac{a_n a_m^* k_m B_{mn} A_{mn}}{\sinh(v_n b)\sinh(v_m b)} \left(\frac{\omega^2}{c^2} - k_n^2 + v_n^2\right) + \\ & \frac{I_b}{I_a} \frac{\pi}{\gamma \beta_{z0}} \times \sum_s \sum_n \sum_m \left\{ \frac{[\omega - v_{z0} k_n] P_{mn}}{\omega - v_{z0} k_n - s\Omega_e/\gamma} \right. \\ & \left. + \frac{\left(\omega^2 v_{\perp 0}^2 / c^2 - v_{\perp 0}^2 k_n^2\right) Q_{mn}}{[\omega - v_{z0} k_n - s\Omega_e/\gamma]^2} \right\} C_{mn} \end{aligned} \tag{4}$$

with

$$\begin{aligned} P_{mn} = & -2 I_s'(v_n r_L) I_s'(v_m r_L) - I_s''(v_n r_L) I_s'(v_m r_L) v_n r_L \\ & - I_s''(v_m r_L) I_s'(v_n r_L) v_m r_L \end{aligned} \tag{5}$$

$$Q_{mn} = I_s'(v_n r_L) I_s'(v_m r_L) \tag{6}$$

$$\begin{aligned} C_{mn} = & \frac{a_n a_m^* k_m B_{mn}}{\sinh(v_n b)\sinh(v_m b)} \left[e^{v_n(b-y_c)} + (-1)^s e^{-v_n(b-y_c)} \right] \\ & \times \left[e^{v_m(b-y_c)} + (-1)^s e^{-v_m(b-y_c)} \right] \end{aligned} \tag{7}$$

where $\beta_{z0} = v_{z0}/c$, $I_a = 4\pi\varepsilon_0 m_e c^3/e$, I_b is the beam current in unit transverse width. r_L represents the Larmor radius, e is the electron charge, $p_\perp$ ($v_\perp$) is the electron transverse velocity and p_z (v_z) is the electron axial velocity. s corresponds to the electron harmonic number. θ represents the electron rotating angle. γ is the relativistic factor and Ω_e is the rest mass cyclotron frequency. I_s is the modified Bessel function of the first kind,

$$A_{mn} = \int_0^b \sinh\left[v_n(b-y)\right]\sinh\left[v_m(b-y)\right]dy \tag{8}$$

$$B_{mn} = de^{j\frac{\pi}{p}(n-m)d}\sin c\left(\frac{\pi}{p}(n-m)d\right)/p$$

Calculation Results

The following case shows the interaction between the cyclotron sheet beam and the fundamental TE mode in the 2-D single grating waveguide. The parameters of the system are considered as follows: b= 2.6mm, p= 3.6mm, d=1.8mm, h=1.6mm, beam voltage V_b= 52.5kV, beam current I_b=100A/m, external magnetic field B_0=2.02T, transverse to axial velocity α=1 and the distance between the grating and beam y_c=1.3mm.

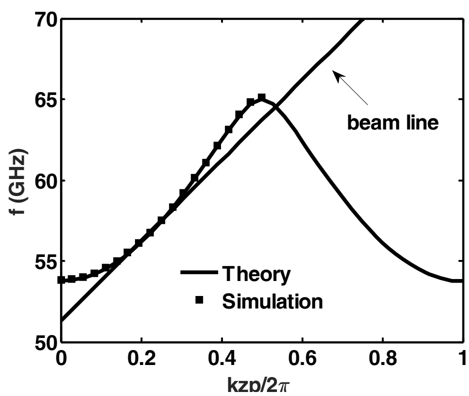

Figure 2. Dispersion curve for the fundamental TE mode.

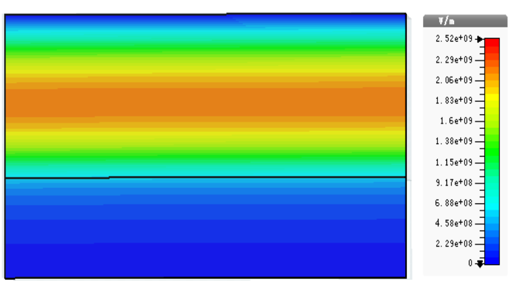

Figure 3. Distribution of electric field in the *y* direction for *z*=0.45p at $\beta p/2\pi$= 0.1.

Figure 2 shows the 'cold' dispersion curve of the fundamental TE mode. It can be seen that the result of the numerical calculation agrees well with the result of simulation, which can be further proved the accuracy of the calculation. The distribution of the electric field in the y-direction for z= 0.45p at □p/2π= 0.1 are displayed in Figure 3. From figure 3, it follows that the electric field amplitude of the middle of the channel is maximal. The electric field amplitude gradually decreases as decaying away from the middle of the channel. Figure 4 demonstrates the effects of the gap distance on the gain for different values of beam current. It can be seen that when y_c increases, the interaction gain increases first and then decreases and it has the same tendency of figure 3, which is in accordance with the laws of physics. In addition, it is evident that the interaction intensity will get stronger with the increasing beam current. It can be observed from figure 5 that the higher velocity ratio will get larger growth rate and the results further verify the gain can be enhanced by increasing the beam current.

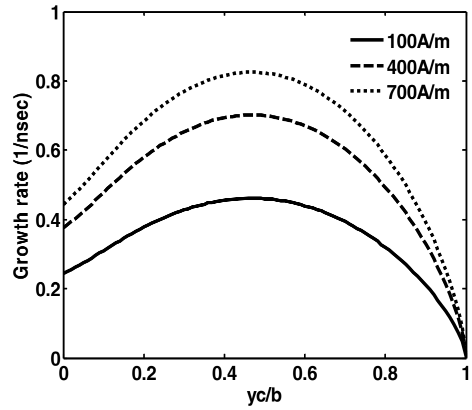

Figure 4. Dependence of interaction gain on gap distance for different values of beam current.

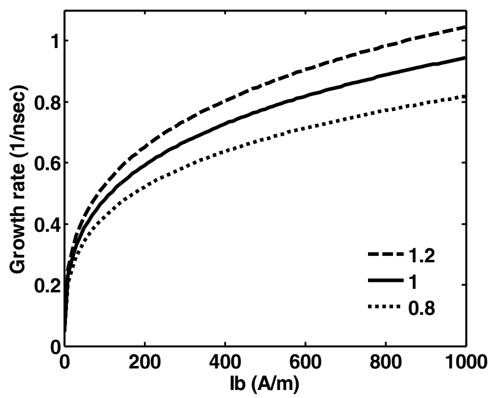

Figure 5. Growth rate versus beam current for different transverse to axial velocity ratios.

Conclusion

In this paper, the numerical calculations of the 'hot' dispersion relation are carried out. For system of single-grating waveguide with cyclotron sheet beam using fundamental TE mode, the gain can be enhanced by putting the beam in the middle of the channel and increasing the current and velocity ratio. Using the TE mode as the operating mode and using this method to investigate are meaningful for the practical design and analysis.

Acknowledgements

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References

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