# A Study on Instabilities of 220 GHz Confocal Waveguide Gyro-TWT

Jie Yang<sup>1,2</sup>, Shouxi Xu<sup>1</sup>, Yong Wang<sup>1,2</sup>, Xiaoyan Wang<sup>1,2</sup>, Lianzheng Zhang<sup>1</sup>

1 Aerospace Information Research Institute, Chinese Academy 2 University of Chinese Academy of Sciences

Beijing, China, 101407

**Abstract**—Instability problem is one of the most significant limits for high power gyro-TWTs. In this paper, the absolute instabilities and backward wave oscillation (BWO) instabilities of a 220 GHz confocal waveguide gyro-TWT are discussed. The results show that the starting current of absolute instabilities is much higher than the operating current 5A, and the critical interaction length of HE05 mode backward wave oscillation is about 10mm.

Keywords- instabilities; confocal waveguide;gyro-TWT

## Introduction

Gyro-TWT can be wildly applied on radar systems, communications and remote sensing because of its broad bandwidth and high power capacity [1]. However, gyro-TWT often suffers from problems with self-oscillations and instabilities. In THz band, the traditional heavily loaded waveguide is not suitable to be the interaction structure. Therefore, confocal waveguide, due to its mode selectivity, is presented. The EM wave can be filtered out from the open sides through diffraction loss and different mode has different loss, which can suppress the mode competition. There are three kinds of instabilities that are important in a gyro-TWT: absolute instabilities, BWO instabilities and reflected power instabilities. The reflective instabilities is caused by outside feedback, so it is difficult to analyze it in theory [2]. In this paper, based on the dispersion relation derived from kinetic theory, the BWO instabilities and absolute instabilities are analyzed.

## **Absolute Instabilities**

As shown in Fig.1, confocal waveguide consists of two mirrors, which have the same aperture width 2a and radius of the curvature  $R_c$ , and the distance between two mirrors  $L_{\perp}$  is equal to  $R_c$ . When the working current is



Fig.1 Structure of confocal waveguide

above the critical threshold for absolute instabilities, the forward wave will couple with backward wave. In addition, the absolute instabilities is a function of the electron beam and EM wave coupling in the gyro-TWT. The absolute instabilities calculation method was well discussed in [3]. The saddle point of the following dispersion equation is the critical condition of absolute instabilities [4].

$$D(\overline{k}_{z},\overline{\omega}) = (\overline{\omega}^{2} - \overline{k}_{z}^{2} - 1)(\overline{\omega} - \beta_{z}\overline{k}_{z} - s\Omega_{c}/\omega_{c})^{2} = -\varepsilon \quad (1)$$

$$D(k_{zs},\overline{\omega}_{zs}) + \varepsilon = 0 \tag{2}$$

$$\frac{\partial D(\bar{k}_{z},\bar{\omega})}{\partial \bar{k}_{z}}\bigg|_{(\bar{k}_{z},\bar{\omega}_{z})} = 0$$
(3)

where  $\overline{\omega} = \omega/\omega_c$  and  $\overline{k_z} = k_z/k_c$  are the normalized frequency and axial wavenumber, respectively;  $\omega_c = k_c c$ ,  $k_c$  is the cutoff number of  $HE_{mn}$  mode and c is the light speed.  $\beta_z$  and  $\beta_t$  are the normalized axial and transverse velocity of electron.  $R_L = \gamma_0 m_e v_t / eB_0$ ,  $\Omega_c = eB_0 / \gamma_0 m_e$ ,  $\gamma_0 = 1/(1 - \beta_t^2 - \beta_z^2)$  are Larmor radius, cyclotron frequency and relativistic factor of electron. When  $B = B_g$  ( $B_g$  is the external magnetic field meets grazing condition), the starting current for absolute instabilities can be written as:

$$I_{c} = \frac{27\beta_{z}^{2}\overline{k}_{zs}^{4}k_{mn}^{2}I_{A}}{4\beta_{t}^{2}\pi} \frac{N_{mn}}{\left\langle \left|F_{mns}\right|^{2}\right\rangle \left[J_{s}'\left(k_{c}r_{L}\right)\right]^{2}}$$
(4)

where  $\overline{\omega}_s = \left\{ b + \left[ 8\beta_z^2 \left( 1 - b^2 \right) + 64\beta_z^4 \right]^{1/2} \right\} / \left( 1 + 8\beta_z^2 \right)$  and

 $\overline{k}_{zs} = (\overline{\omega}_{s} - b)/4\beta_{z} \quad \text{And} \quad I_{A} \text{ is the Alfven current,}$  $N_{mn} = \int_{S} |\psi(x, y)|^{2} dS \quad \text{,} \quad F_{mns}(x, y) = \left[\frac{1}{k_{c}}\left(\frac{\partial}{\partial x} + j\frac{\partial}{\partial y}\right)\right]^{s} \psi(x, y) \quad \text{,}$ 

where 
$$\psi(x, y)$$
 is the membrane function of  $HE_{mn}$  mode.



Fig.2 Critical current for absolute instability versus voltage



A conclusion can be made from Fig.2 that, increasing the voltage can increase the starting current of absolute instability. As shown in Fig.3, a higher pitch factor can lead to a lower critical current of absolute instability. Moreover, Fig.2 and Fig.3 can indicate that by reducing the magnetic field of the interaction structure, a higher critical current of absolute instability can be reached.

### **BWO** Instabilities

When the cyclotron beam mode synchronous with a backward waveguide mode, the BWO instabilities will occur. By replacing  $\overline{\omega}$  with  $\overline{\omega}_0 + \delta \overline{\omega}$  and  $\overline{k}_z$  with  $\overline{k}_{z0} + \delta \overline{k}_z$ , and  $|\delta \overline{\omega}/\overline{\omega}| \square 1$ ,  $|\delta \overline{k}_z/\overline{k}_{z0}| \square 1$ , the dispersion relation can be rewritten as follows[5]:

$$g(\delta\overline{\omega}) = \delta\overline{\omega}^3 - \varepsilon_1 \delta\overline{\omega} + \varepsilon_2 = 0 \tag{5}$$

where 
$$\varepsilon_1 = \frac{27}{4} \left(\frac{\pi}{2L}\right)^2 \frac{\beta_z^2 \overline{v}_g^2}{\left(\beta_z + \overline{v}_g\right)^2}, \ \varepsilon_2 = \frac{27I_c K_m}{8\overline{\omega}_0} \frac{\beta_z \overline{v}_g^2}{\left(\beta_z + \overline{v}_g\right)^3}$$

$$\overline{v}_{g} = -\overline{k}_{z0}/\overline{\omega}_{0}$$
 and  $K_{m} = \frac{-4\pi\beta_{t0}^{2}}{\gamma_{0}\beta_{z}I_{A}k_{c}^{2}} \frac{\langle |F_{mns}|^{2} \rangle}{N_{mn}^{2}} \left[ J_{s}'(k_{c}r_{L}) \right]^{2}$ 

And the instability occurs when  $\varepsilon_1^3 = \frac{27}{4}\varepsilon_2^2$ , so the critical oscillation start current can be expressed as:

$$I_{st} = 2\bar{\omega}_0 \left(\frac{\pi}{2L}\right)^3 \beta_z^2 \bar{\nu}_g \frac{1}{K_m} \tag{6}$$



Fig.4 BWO Start current versus interaction length

The start current is proportional to  $1/L^3$ , where *L* is the length of interaction length. When the starting current is 5 A, the critical interaction length is 10mm.

# Summary

In this paper, the absolute instabilities and BWO instabilities of a 220 GHz confocal waveguide gyro-TWT has been investigated based on kinetic theory. The critical current of absolute instabilities is 10A that is higher than the operating current. In addition, the interaction length should less than 10mm to avoid  $HE_{05}$  mode BWO instabilities.

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#### References

- K. R. Chu, "Overview of research on the gyrotron traveling-wave amplifier," IEEE Trans. Plasma Sci., vol. 30, no. 3, pp. 903–908,Jun. 2002
- C. D. Joye, "A novel wideband 140 GHz gyrotron amplifier," Cambridge, MA, USA: Massachusetts Inst. Technol., 2008
- Y.Y. Lau, K. R. Chu, L R Barnett, et al. "Gyrotron travelling wave amplifier: I. Analysis of oscillations" [J]. International Journal of Infrared and Millimeter Waves, 1981, 2(3): 373-393
- W. Sun, S. Yu, Z. Wang and Y. Yang, "Linear and Nonlinear Analyses of a 0.34-THz Confocal Waveguide Gyro-TWT," in IEEE Transactions on Plasma Science, vol. 46, no. 3, pp. 511-517, March 2018.
- Shih Hung. Chen, Liu. Chen, "Linear and nonlinear behaviors of gyrotron backward wave oscillators". Physics of Plasmas. 19. 10.1063/1.3688892. Feb 2012