# Study on Beam-shaping Mirrors Based on Gaussian Beam Propagation Theory

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**Abstract:** Based on the Gaussian beam propagation theory, the phase correcting of five Gaussian beams with different parameters is studied. A numerical model including two phase correcting mirrors is established to correct the phase of incident beam. The purpose of these mirrors is to eliminate the astigmatism and ellipticity of the incident Gaussian beams, compensate the phase of the Gaussian beams on the correcting mirror, and obtain the ideal Gaussian beam on the ouput window. After correcting, the vector correlation coefficient between the output Gaussian beams and the ideal Gaussian beams are above 99.1%.

**Keywords:** Gaussian beam; phase correcting; Gyrotron; quasi-optical mode converter

### Introduction

Gaussian beam is a very important spatial distribution form of electromagnetic field. The Gaussian beam refers to the electromagnetic beam whose transverse amplitude distribution is a Gaussian function. The beam radiated from the quasioptical launcher and propagating in the mirror system of the gyrotron quasi optical mode converter is a Gaussian-like beam. It is important to study the transformation characteristics of Gaussian beam for designing the mirror system of mode converter. The general Gaussian beam has astigmatism and ellipticity. Ellipticity is defined as the ratio of the beam waists in perpendicular directions, and astigmatism is defined as the difference of the positions of the beam waists[1,2]. The radiation beam of quasi-optical launcher has a large ellipticity and astigmatism, and the energy diverges with the increase of propagation distance. Although the ellipticity and astigmatism of the beam can be adjusted by using the mirrors with regular surface, such as plane mirrors and quadratic surface mirrors, but it is difficult to eliminate the astigmatism of the beam, and the wave-front phase of the output beam is not on the same plane[3-5]. In this paper, based on the propagation theory of Gaussian beam, the propagation of five Gaussian beams with different beam parameters at 170GHz along the Y-axis is studied. The beam is incident on mirror 1 along the Y-axis at an incident angle of 45°. After being reflected by mirror 1, it is incident on mirror 2 at an incident angle of 45°. Then it is

reflected by mirror 2, and propagating along the Y-axis to the output plane. The numerical simulation model is shown in Fig. 1.





## Theory of the Beam-shaping Mirrors

The expression of the Gaussian beam propagating along the Yaxis can be written as follows

$$u(x, y, z) = \frac{1}{\sqrt{w_x w_z}} e^{-\frac{(x - x_0)^2}{w_x^2} - \frac{(z - z_0)^2}{w_z^2}} e^{j\Phi}$$
(1)

$$w_{x} = w_{0x} \sqrt{1 + \left(\frac{2(y - y_{0x})}{kw_{0x}^{2}}\right)^{2}}$$
(2)

$$w_{z} = w_{0z} \sqrt{1 + \left(\frac{2(y - y_{0z})}{kw_{0z}^{2}}\right)^{2}}$$
(3)

$$\Phi = -ky - \phi_{0x} - \phi_{0z} + \frac{k(x - x_0)^2}{2R_x} + \frac{k(z - z_0)^2}{2R_z}$$
(4)

$$\phi_{0x} = \arctan\left(\frac{2(y - y_{0x})}{kw_{0x}^2}\right) \tag{5}$$

$$\phi_{0z} = \arctan\left(\frac{2(y - y_{0z})}{kw_{0z}^2}\right) \tag{6}$$

$$R_{x} = (y - y_{0x}) + \frac{1}{(y - y_{0x})} \left(\frac{kw_{0x}^{2}}{2}\right)^{2}$$
(7)

$$R_{z} = \left(y - y_{0z}\right) + \frac{1}{\left(y - y_{0z}\right)} \left(\frac{kw_{0z}^{2}}{2}\right)^{2}$$
(8)

where k is the wave number in free space,  $y_{0x}$  and  $y_{0z}$  is the positions of the beam waists  $w_{0x}$  and  $w_{0z}$ , respectively.  $(x_0, z_0)$  is the beam center in x-z plane of the incident beam. The surface of mirror 1 can be described as

$$S(x, y, z) = \frac{(\phi_{out} - \phi_{cout}) - (\phi_{in} - \phi_{cin})}{2k \cos \alpha}$$
(9)

where  $\phi_{out}$  and  $\phi_{cout}$  can be obtained by solving equations, and the specific derivation process is referred to [1]. *S* is the deformation of the mirror surface and  $\alpha$  is the incident angle of the beam on M1 and M2. In this paper the incident angle is 45°.

## **Results of the Simulation**

The parameters of the mirrors are shown in Table I. The simulation results are shown in Table II. Take the fifth group of data in Table II as an example. Fig.2 shows the field distribution(dB) and phase pattern on output plane before mirrors design. The field distribution(dB) and phase pattern on output plane after mirrors design are shown in Figure 3. It is obviously that the field on the output plane is almost a Gaussian beam field distribution.

Table 1. The parameters of the mirrors.

	Mirror	Mirror	Tilt
	center(mm)	size(mm)	angle(deg)
M1	(0,300,0)	150×150	$\theta_x = 0,  \theta_z = 45$
M2	(0,300,300)	150×150	$\theta_x = 0,  \theta_z = 45$
Output plane	(0,600,300)	100×100	$\theta_{1}=0,  \theta_{2}=0$

Waist radius in two directions and astigmatis- m of incident Gaussian beams (mm)	Waist radius of target Gaussi -an beams in two directi- ons (mm)	scalar and vector correlation coefficient before correcting (%)	scalar and vector correlation coefficient after correcting (%)
20,25,50	28,28	99.23,82.41	97.77,94.00
20,25,50	26,26	97.38,82.38	99.39,99.22
20,24,50	22,22	93.98,79.06	99.70,99.03
20,24,500	22,22	96.69,86.91	99.66,99.09
20.24.800	22.22	96.57.89.45	99.69.99.08

**Table 2.** The results of the simulation







**Figure 3.** (a) Field amplitude (dB) distribution (b) phase pattern distribution on the window after phase correcting.

### Conclusion

Two mirrors are designed to correct the Gaussian beams with different parameters at the frequency 170GHz. Gaussian beams with astigmatism and ellipticity are transformed into Gaussian beams without astigmatism and ellipticity on the output plane after being reflected by the mirror 1 and mirror 2. The method in this paper can provide a basis for designing the phase correcting mirror system of gyrotron quasi-optical mode converter. This method consumes less calculation time, and it is especially suitable for optimizing the parameters of mirror system.

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