# Analysis of High Frequency Characteristics for a Meander Line Slow-wave Structure

Zheng Wen<sup>1,2</sup>, Jirun Luo<sup>1,2</sup>, Yu Fan<sup>1,2</sup>, Chen Yang<sup>1,2</sup>, Fang Zhu<sup>2</sup>, Min Zhu<sup>2</sup>, Wei Guo<sup>2</sup>, Yubin Gong<sup>3</sup>, and Jinjun Feng<sup>4</sup>

<sup>1</sup>Key Laboratory of High Power Microwave Source and Technologies, Institute of Electronics, Chinese Academy of Sciences, Beijing 100190, China <sup>2</sup>University of Chinese Academy of Sciences, Beijing 100049, China <sup>3</sup>National Key Laboratory of Science and Technology on Vacuum Electronics, University of Electronic Science and Technology of China, Chengdu, 610054, China <sup>4</sup>Vacuum Electronics National Laboratory, Beijing Vacuum Electronics Research Institute, Beijing, 100016, China

# Contact Author Email: wenzheng17@mails.ucas.edu.cn

**Abstract:** Analysis of high frequency characteristics for a meander line slow-wave structure (ML-SWS) has been carried out by field-matching methods with the dyadic Green's function. The effect from thickness of the meander line was considered and the theoretical results had been compared with the simulation ones.

**Keywords:** Meander line; slow-wave structure; traveling wave tube (TWT); dyadic Green's function; field-matching methods.

### Introduction

Owing to the traditional slow-wave structure (SWS), the core of the TWT, is difficult to be fabricated by conventional fabrication technologies, with the operation frequency moving to Ka-band or above [1]. A new type of the SWS, the meander line slow-wave structure, compatible with the microfabrication technologies, comes into being [2]. To the best of our knowledge, the most analyses of the ML-SWS are exclusively with numerical simulations [2-3], and the few theoretical analyses are based on ignoring the thickness of the ML [4-5].

## Model and Results

The ML-SWS consists of the meander line and Rectangular waveguide as shown in figure 1. For lossless system, the model can be simplified with just one period of the ML-SWS, according to Fouquet's theorem. The electrical field as source can excite the magnetic fields,  $\overline{H}_{RW}$  in the rectangular waveguide, and  $\overline{H}_{ML}$  in the meander line, respectively. They satisfy the field-matching equation as:

$$\vec{H}_{RW}(\vec{R}) = \vec{H}_{ML}(\vec{R}), \vec{R} \in \text{Meander-Line}$$
(1)

The magnetic field can be described with the equivalent electrical field by the following equation:

$$\vec{H}\left(\vec{R}\right) = -i\omega\varepsilon_0 \int_{S^*} \vec{\overline{G}}\left(\vec{R}, \vec{R'}\right) \cdot \left(\hat{n} \times \vec{E}\left(\vec{R'}\right)\right) dS' \qquad (2)$$

where  $\vec{E}(\vec{R}^{i})$  is the equivalent electrical fields and  $\vec{G}(\vec{R},\vec{R}^{i})$  is the dyadic Green's function, and the  $\vec{E}(\vec{R}^{i})$  can be written with sets of Ritz basic functions  $\{\varphi_n\}$ , which are shown as below:

$$\vec{E} = \sum_{n} A_{n} \varphi_{n} \tag{3}$$



**Figure 1.** Geometry of the ML-SWS. (a) 3D model view, (b) front view, (c) top view.

The Ritz basic functions  $\{\varphi_n\}$  can be obtained according to [6]. For instance, the  $\varphi_n$  in x-axis direction can be written:

$$\varphi_n = \hat{x} \cos\left(n\pi \left(\underline{x} - b_0\right) / (b_1 - b_0)\right) \tag{4}$$

The dyadic Green's function  $\vec{G}$  can be divided into two part,  $\vec{G}_{T}$  for the sum of "TE" and "TM" modes, and  $\vec{G}_{TEM}$  for the "TEM" mode.

$$\vec{\vec{G}} = \vec{\vec{G}}_T + \vec{\vec{G}}_{TEM}$$
(5)

where  $\vec{\vec{G}}_T$  and  $\vec{\vec{G}}_{TEM}$  can be written as following:

$$\vec{\overline{G}}_{T} = \frac{1}{V_{\text{out}}} \sum_{\kappa} \left( \frac{\left(2 - \delta_{k_{c}}\right) \left( \nabla \times \left(\vec{C} \cdot f_{M}(x, y, z)\right) \\ \left[\nabla \times \left(\vec{C} \cdot f_{M}(x', y', z')\right)\right]^{*}\right)}{\left(k_{c}\right)^{2} \left((\kappa)^{2} - k^{2}\right)} \\ + \frac{\left(2 - \delta_{k_{c}}\right) \left(\nabla \times \nabla \times \left(\vec{C} \cdot f_{N}(x, y, z)\right) \\ \left[\nabla \times \nabla \times \left(\vec{C} \cdot f_{N}(x', y', z')\right)\right]^{*}\right)}{\left(k_{c}\right)^{2} \left(\kappa\right)^{2} \left(\kappa\right)^{2} - k^{2}\right)} \right)$$
(6)

$$\vec{G}_{TEM} = \begin{cases} \frac{3\sum_{s} \frac{1}{(h_{0}^{2} - k^{2})} \left(\hat{x}(x - x_{s}^{l})e^{ih_{0}z}\right) \left(\hat{x}(x - x_{s}^{l})e^{-ih_{0}z^{l}}\right)}{-\hat{y}(y - y_{s}^{l})e^{ih_{0}z}} \left(\hat{y}(y - y_{s}^{l})e^{-ih_{0}z^{l}}\right)}{2V_{l}\left(\left(x_{l}^{l} - x_{s}^{l}\right)^{2} + \left(y_{l}^{l} - y_{s}^{l}\right)^{2}\right)}{3\sum_{s} \frac{1}{(h_{0}^{2} - k^{2})} \left(\hat{-\hat{x}}(x - x_{s}^{l})e^{ih_{0}y}\right) \left(\hat{-\hat{x}}(x - x_{s}^{l})e^{-ih_{0}y^{l}}\right)}{\hat{z}(z - z_{s}^{l})e^{-ih_{0}y^{l}}}\right)} \end{cases}$$
(7)

Hence, considering the superposition principle, the fieldmatching equation can be rewritten with the matrix form:

[Y][A] = 0 (8) where the matrix [A] is a set of variables  $\{A_n\}$ . In fact, it is also a vector. The matrix [Y] can be expressed by variables  $\omega$  and  $\theta$ . The nonzero solution exists only if the value of corresponding determinant |Y| is zero.

Accordingly, the dispersion characteristic equation is obtained as following:

$$|Y| = 0 \tag{9}$$

In addition, according to equation (8), the variable  $A_n$  can be solved for concrete frequency  $\omega_0$  and phase shift  $\theta_0$ , namely,  $A_n|_{\omega_0,\theta_0}$ . Considering no exciting source, the electro-magnetic field in the ML-SWS can be obtained:

$$\begin{cases} \left. \overrightarrow{H}_{\text{out}}\left(\overrightarrow{R}\right) \right|_{\omega_{0},\theta_{0}} = -i\omega_{0}\varepsilon_{0}\sum_{n} \left( A_{n} \right|_{\omega_{0},\theta_{0}} \int_{S^{*}} \overrightarrow{G}\left(\overrightarrow{R},\overrightarrow{R}^{*}\right) \cdot \left(\widehat{n} \times \varphi_{n}\right) dS^{*} \right) \\ \left. \left. \overrightarrow{E}_{\text{out}}\left(\overrightarrow{R}\right) \right|_{\omega_{0},\theta_{0}} = -\left( \nabla \times \overrightarrow{H}_{\text{out}}\left(\overrightarrow{R}\right) \right|_{\omega_{0},\theta_{0}} \right) \middle/ \left( i\omega_{0}\varepsilon_{0} \right) \end{cases}$$
(10)

The interaction impedance  $R_{\omega_0,\theta_0}$  can be calculated:

$$R_{\omega_0,\theta_0} = \left| \vec{E}_{\text{out}} \left( \vec{R} \right) \right|_{\omega_0,\theta_0} \right|^2 / \left( 2\beta_{\omega_0,\theta_0}^2 P \right)$$
(11)

where *P* is the total power,  $\beta_{\omega_0,\theta_0} = (\theta_0 + 2s\pi)/T$ , *T* is the length of a period.

Here, an example is calculated, and the parameters of the ML-SWS are exhibited in Table 1:

Table 1. The parameters of the ML-SWS (mm)



**Figure 2.** Comparison of dispersion of a ML-SWS. As shown in figure 2. The dispersion results carried out by the proposed method were compared with those simulated with HFSS code, the relative errors between them is under 7%, which demonstrates that the theoretical results and simulated ones are in good agreement.

In figure 2 and 3, the dispersion of mode 1 is very close to that of mode 2 at point A. As shown in figure 3. When  $z_2=1.6$  mm and  $z_2=2.0$  mm, there is obvious band gap between the dispersions of adjacent two modes. When  $z_2=2.3$  mm, the corresponding structures and sizes of two half-period of the ML are same, and dispersions of two modes are very close to each other near point A, i.e., the band gap seems to disappear.



Figure 3. Effect of a parameter  $z_2$  on the dispersion of the ML-SWS.

#### Conclusions

For analyzing the high frequency characteristics of the ML-SWS, the field-matching method with the dyadic Green's function has been utilized in this paper. The theoretical and HFSS simulated results were compared, the relative errors in presented range are under 7%. The band gap of the dispersions may disappear when the corresponding structures and sizes of two half-period of the ML are same for the adjacent two slow wave modes. The more results and analysis will be presented on the conference.

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