The Hot Dispersion Equation of the Backward Wave Oscillator with the Nonuniform Grating

Fengzhen Zhang
[1]Key Laboratory of Science and Technology on High Power Microwave Sources and Technologies
Aerospace Information Research Institute, Chinese Academy of Sciences
[2] School of Electronic, Electrical and Communication Engineering
University of Chinese Academy of Sciences
Beijing, China
zhangfengzhen16@mails.ucas.ac.cn

Zhaochuan Zhang
[1]Key Laboratory of Science and Technology on High Power Microwave Sources and Technologies
Aerospace Information Research Institute, Chinese Academy of Sciences
Beijing, China
zczhang@mail.ie.ac.cn

Dongping Gao
[1]Key Laboratory of Science and Technology on High Power Microwave Sources and Technologies
Aerospace Information Research Institute, Chinese Academy of Sciences
Beijing, China
dongpinggao@mail.ie.ac.cn

Xiaoyan Wang
[1]Key Laboratory of Science and Technology on High Power Microwave Sources and Technologies
Aerospace Information Research Institute, Chinese Academy of Sciences
[2] School of Electronic, Electrical and Communication Engineering
University of Chinese Academy of Sciences
Beijing, China
wangxiaoyanns@163.com

Abstract: *The hot dispersion equation of the backward wave oscillator (BWO) with the partially dielectric-loaded (PDL) nonuniform grating is proposed in this paper. The expression of the hot dispersion equation is given and the resonant growth rate is verified by the Pierce small signal gain of a BWO with the PDL nonuniform grating.*

Keywords: Nonuniform grating, Resonant growth rate, Slow wave structure, Backward wave oscillator

Introduction

The nonuniform grating has been used in the BWOs and clinotrons[1,2]. The ohmic losses of the nonuniform grating are lower and the hybrid space-surface waves of the slow wave structure (SWS) reduce the starting current of the BWO with the nonuniform grating. It is important to study the high-frequency characteristics and beam-wave interaction of the PDL nonuniform-grating-based SWS. The dispersion curve and small signal gain have been discussed in [3]. The hot dispersion equation which can describe the beam-wave interaction is derived and is verified in this paper.

Model and Formula

The positional relationship between the sheet beam and the PDL nonuniform-grating-based SWS is shown in Fig. 1. The number of grooves per period is *N*, the width of the grooves is *d*, the distance between grooves is *l*, the length per period is *L*, the distance between grating surface and the upper wall is *D* and the width of the gratings is *a*. For the *p*th groove, the depth of the groove is h_p, the thickness of the dielectric is h_{p1} and the relative permittivity of the dielectric is ε_p. The width of the beam is *a*, the thickness of the beam is d_e, the beam-grating distance is b, the volume density of charge is $\rho_0 = I_e / S$ and I_e is the current of the beam.

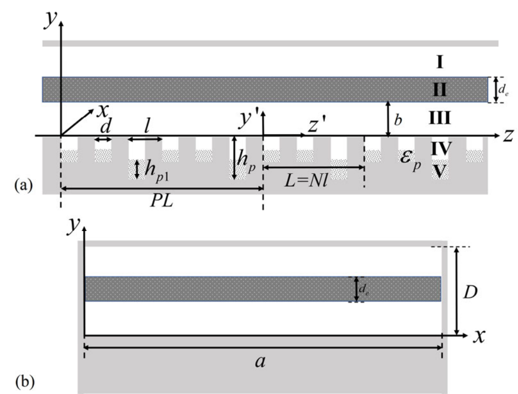

Fig. 1. Model of sheet beam and PDL nonuniform-grating-based SWS: the light gray shadowing, the deep gray shadowing and the black shadowing indicate metal, dielectric and sheet beam, respectively. (a) Axial cross-section view; (b) transversal cross-section view.

The process of solving the hot dispersion equation is similar to that of solving the dispersion equation for the PDL nonuniform-grating-based SWS[3]. The area is divided into five regions: $b + d_e < y \le D$ is the region between the beam and the upper wall of the waveguide, $b < y \le b + d_e$ is the region of the beam, $(0 < y \le b)$ is the region between the beam and the grating surface, $\left(-h_p + h_{p1}\right) \le y \le 0$ is the region without dielectric in the groove, and $-h_p \le y < \left(-h_p + h_{p1}\right)$ is the region with dielectric in the groove.

Through a detailed algebraic operation, we can obtain the hot dispersion equation as follows:

$$\sum_{s=-\infty}^{s=\infty} \sum_{n=-\infty}^{n=\infty} \left(\delta_{ns} - \sum_{m=0}^{m=\infty} \sum_{p=1}^{p=N} X_{ms} Y_{nmp} \right) A_{1n} = 0 \qquad (1)$$

This work was supported by the National Defense Pre-Research Foundation of China under Grant JZX2017-1470/Y364

$$X_{ms}Y_{nmp}=\frac{\left(G(0)+J_p g(0)\right)W\left(\beta_s,\frac{m\pi}{d}\right)e^{j\beta_s(p-q)l}}{\left(F_s'(0)+T_s f_s'(0)\right)L}\times\frac{2\left(F_n(0)+T_n f_n(0)\right)W\left(-\beta_n,\frac{m\pi}{d}\right)e^{-j\beta_n(p-q)l}}{\left(G'(0)+J_p g'(0)\right)\left(1+\delta_{m0}\right)d} \tag{2}$$

$$R_n=\frac{R_n^1-R_n^2}{R_n^3-R_n^4}\qquad T_n=\frac{T_n^1-T_n^2}{T_n^3-T_n^4} \tag{3}$$

$$R_n^1=\left(\varepsilon_e k_0^2-k_x^2\right)O'\left(b+d_e-D\right)U\left(b+d_e\right) \tag{4}$$

$$R_n^2=\left(k_0^2-k_x^2\right)O\left(b+d_e-D\right)U'\left(b+d_e\right) \tag{5}$$

$$R_n^3=\left(k_0^2-k_x^2\right)O\left(b+d_e-D\right)u'\left(b+d_e\right) \tag{6}$$

$$R_n^4=\left(\varepsilon_e k_0^2-k_x^2\right)O'\left(b+d_e-D\right)u\left(b+d_e\right) \tag{7}$$

$$T_n^1=\left(k_0^2-k_x^2\right)\left(U'(b)+R_n u'(b)\right)F(b) \tag{8}$$

$$T_n^2=\left(\varepsilon_e k_0^2-k_x^2\right)\left(U(b)+R_n u(b)\right)F'(b) \tag{9}$$

$$T_n^3=\left(\varepsilon_e k_0^2-k_x^2\right)\left(U(b)+R_n u(b)\right)f'(b) \tag{10}$$

$$T_n^4=\left(k_0^2-k_x^2\right)\left(U'(b)+R_n u'(b)\right)f(b) \tag{11}$$

$$J_p=\frac{k_{\tau 2}^2 I\left(h_{p1}\right)G'\left(h_{p1}-h_p\right)-k_{\tau 3}^2 I'\left(h_{p1}\right)G\left(h_{p1}-h_p\right)}{k_{\tau 3}^2 I'\left(h_{p1}\right)g\left(h_{p1}-h_p\right)-k_{\tau 2}^2 I\left(h_{p1}\right)g'\left(h_{p1}-h_p\right)} \tag{12}$$

$$O(y)=\begin{cases}\cosh\left(\tau_{yn}y\right) & k_x^2+\beta_n^2-k_0^2>0\\ \cos\left(\tau_{yn}y\right) & k_x^2+\beta_n^2-k_0^2<0\end{cases} \tag{13}$$

$$O'(y)=\begin{cases}\tau_{yn}\sinh\left(\tau_{yn}y\right) & k_x^2+\beta_n^2-k_0^2>0\\ -\tau_{yn}\sin\left(\tau_{yn}y\right) & k_x^2+\beta_n^2-k_0^2<0\end{cases} \tag{14}$$

$$\varepsilon_e=1-\frac{\omega_p^2}{\gamma^3\left(\omega-\beta_n v_0\right)^2} \tag{15}$$

$$\det\{\boldsymbol{I}-\boldsymbol{XY}\}=0 \tag{16}$$

Equation (16) is the hot dispersion equation of the PDL nonuniform-grating-based SWS. Given *f*, the corresponding complex propagation coefficient β_n can be obtained. The imaginary part of β_n is the resonant growth rate[4].

$$G=8.686\,\mathrm{Im}\left(\beta_n\right)L\ \left(\text{dB/period}\right) \tag{17}$$

Theoretical Calculation

The parameters of the SWS are the same as those in [3]. The beam-grating distance is $b=10$ μm, the thickness of the beam is $d_e=70$ μm, the current of the beam is $I_e=50$ mA and the interaction length is $L_c=100L$. The resonant growth rates and Pierce small signal gains of the −3rd spatial harmonic of the TE_{10} mode and TE_{20} mode are calculated and are shown in Fig. 2. The trend of the resonant growth rates calculated by the hot dispersion equation is the same as the trend of the gains calculated by the Pierce small signal gain, which shows the correctness of the hot dispersion equation.

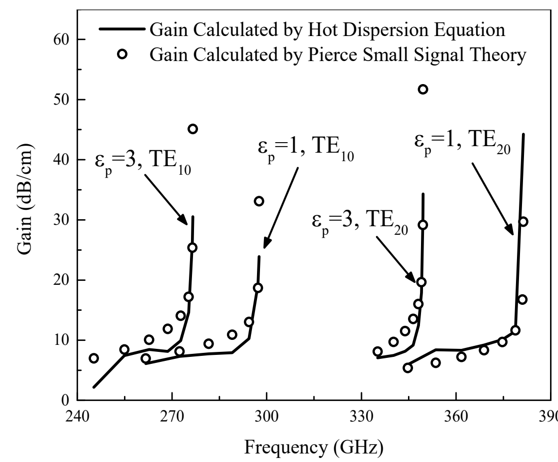

Fig. 2. The resonant growth rates calculated by the hot dispersion equation and the gains calculated by the Pierce small signal gain.

Conclusion

In this paper, the hot dispersion equation of the BWO with the PDL nonuniform grating is derived. The hot dispersion equation is verified by the Pierce small signal gain and the resonant growth rates calculated by the hot dispersion equation are in good agreement with the gains calculated by the Pierce small signal gain.

References

[1] E. M. Khutoryan, Y. S. Kovshov, A. S. Likhachev, S. S. Ponomarenko, S. A. Kishko, et al., “Excitation of Hybrid Space-Surface Waves in Clinotrons with Non-uniform Grating,” *J Infrared Milli Terahz Waves*, vol. 39, no. 3, pp. 236–249, Nov. 2017, 10.1007/s10762-017-0453-3.

[2] S. S. Ponomarenko, S. A. Kishko, E. M. Khutoryan, A. N. Kuleshov and B. P. Yefimov, “The extension of the operation frequency range of the resonant BWOs by use of the multistage gratings,” in *International Conference on MMET*, Dnipropetrovsk, Ukraine, 2014, pp.233-236.

[3] F. Zhang, Z. Zhang, D. Gao, and X. Wang, "Analysis of the Small Signal Gain of a Sheet Beam BWO With a Partially Dielectric-Loaded Nonuniform Grating," *IEEE Trans. Electron Devices,* vol. 66, no. 9, pp. 4022-4028, Sep. 2019. doi: 10.1109/TED.2019.2925911.

[4] M. Cao, W. Liu, Y. Wang and K. Li, “Three-dimensional theory of Smith-Purcell free-electron laser with dielectric loaded grating,” *J. Appl. Phys*, vol. 116, no. 10, Sep. 2014. Accessed on: September, 11, 2014, DOI: 10.1063/1.4894706, [Online]