

# Transitions in Electron Emission and Gas Breakdown from Nanoscale to Microscale

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**Abstract:** *The miniaturization of electronic devices requires a deeper understanding of electron emission and gas breakdown dynamics at nano- and microscales. Micro- and nanoelectromechanical systems (MEMS and NEMS, respectively) for sensing and scanning, microplasma applications, and directed energy devices require an accurate characterization of electron emission behavior for accurate predictions of device behavior. While breakdown has historically been predicted by Paschen's law—driven by Townsend avalanche processes—this fails as gap distance decreases to the point where field emission (FE) become relevant. Further decreasing gap distance makes electron emission space-charge-limited as defined by the Child-Langmuir law (CSCL) at vacuum, the Mott-Gurney law (MG) if collisions contribute, or quantum space charge behavior (QSCL) at nanoscales when Schrodinger's wave equation must be used to consider single particle effects. This study nondimensionalizes the governing equations defining these underlying phenomena to identify transition points between them and understand the underlying physics dominating the emission behavior in these regimes to aid device design and experimental setup. A sample case theorizes a nexus occurring between all emission mechanisms at a pressure of approximately 1780 Torr, a gap distance of 62 nm, and an applied voltage of 6 V. Additionally, we discuss the implications of work function and surface roughness on these behaviors.*

**Keywords:** microdischarge; breakdown; microscale; plasma

## Introduction

Electronic devices trending towards smaller scales necessitates a greater understanding of electron emission and gas breakdown behavior at these scales. Paschen's law (PL) [1] has traditionally predicted gas breakdown considering electron avalanche behavior, but is insufficient at microscale and smaller gaps when field emission (FE) causes a further reduction of breakdown voltage with decreasing gap distance [2,3] rather than an increase as predicted by PL to the left of the Paschen minimum. Modified PL models incorporating FE into PL describe the coupled FE/TA regime [4-7]. Further reducing gap distance yields the space-charge-limited regime (SCL) [8,9] where the injection current is no longer able to be increased. An additional decrease in gap distance results in a regime where single particle effects must be considered through Schrödinger's equation [10]. A unified emission model encompassing all of these emission and breakdown mechanisms is vital for accurately

characterizing electron behavior and understanding device behavior at these scales.

## Model Development

To set up the model we outline the governing equations guiding each regime. Paschen's law, representing the Townsend regime, is given by [1]

$$V = \frac{Bpd}{\ln(Apd) - \ln[\ln(1 + \gamma_{SE}^{-1})]} \quad (1)$$

with  $V$  giving the breakdown voltage,  $p$  the gas pressure,  $d$  the gap distance,  $\gamma_{SE}$  the secondary emission coefficient, and  $A$  and  $B$  gas-dependent parameters. Next, the coupled regime describing FE/TA is given by [5]

$$\frac{2E^2 v_d \epsilon_0 \{1 - \gamma_{SE} [\exp(\alpha d) - 1]\}}{D_{FN} d j_{FN} [\exp(\alpha d) - 1]} = \frac{\exp(x_0) (1 + 2\bar{E} x_0)}{x_0} \quad (2)$$

where  $E$  is the breakdown field,  $\alpha$  is the ionization coefficient,  $v_d$  is the drift velocity,  $j_{FN}$  is the Fowler-Nordheim current,  $D_{FN}$  is a field emission parameter, and  $x_0$  is a breakdown field-dependent term. The transitions between FN and the SCL regime are described by Poisson's equation with

$$\frac{d^2 V}{dx^2} = \frac{J}{\epsilon_0 v'} \quad (3)$$

where  $x$  is position,  $J$  is the current density, and  $v$  is the electron velocity, and a force balance including mobility with

$$m \frac{dv}{dt} = e \frac{dV}{dx} - \frac{ev}{\mu}, \quad v_d = \mu E, \quad (4)$$

where  $m$  is the electron mass,  $e$  is the electron charge, and  $\mu$  is the electron mobility. Finally, the region where single-particle effects must be considered is described by simultaneously solving Schrodinger's 1D wave equation given by [10]

$$\frac{\hbar^2}{2m} \frac{d^2 \psi}{dx^2} - eV\psi = U\psi, \quad (5)$$

where  $\psi$  is the wave function and  $U$  is the electron injection energy and the current density described by

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$$J = e \left( \frac{i\hbar}{2m} \right) [\psi\psi^{*'} - \psi^*\psi'], \quad (6)$$

where ' is a position-dependent derivative and \* is the complex conjugate. Equations (1)-(6) describe the electron emission behavior from nanoscale up to the traditional PL. We derived a set of consistent scaling parameters to nondimensionalize the governing equations to identify the transition points between them. Table 1 defines the scaling parameters and definitions that are coupled with (1)-(6) to describe the dimensionless mode. While some of the scaling parameters have a physical meaning (e.g. pressure scaling with electric field and material constant), most of the scaling parameters were derived based on the mathematical forms of the equations. This set of scaling parameters eliminates the most material dependences, resulting in a breakdown model that is universal up to PL.

TABLE I. SCALING PARAMETERS FOR NONDIMENSIONALIZING (1)-(6)

Scaling Parameter	Equation
Pressure	$p_* = E_*/B_p$ [Torr]
Electric Field	$E_* = D'_{FN} = 0.95B_{FN}\phi_*^{3/2}$ [V/m]
Current Density	$j_0 = C'_{FN}E_*^2 = \frac{A_{FN}}{\phi_*t^2(y)}E_*^2$ [A/m <sup>2</sup> ]
Temperature	$T_* = \frac{\pi m_g \sigma_{CE}}{2ekB_p} \left[ \frac{C'_{FN}D'_{FN}L^2 A_p p_*}{2\epsilon_0} \right]^2$ [K]
Length	$L = \left[ \frac{\hbar}{2m_e e E_*} \right]^{1/3}$ [m]
Time	$\tau_*^2 = \frac{m_e L}{e E_*}$ [s <sup>2</sup> ]
Mobility	$\mu_* = \frac{e\tau_*}{m_e}$ [m <sup>2</sup> /Vs]
Work Function	$\phi_*^{1/2} = \left[ \frac{2A_{FN}^2 m_e \hbar (0.95B_{FN})}{(\epsilon_0 t^2(y))^3 e^2} \right]^{1/3}$ [eV <sup>1/2</sup> ]
Wave Function	$\psi_*^2 = \epsilon_0 E_*/eL$ [m <sup>-3</sup> ]
Electron Energy	$U_* = eV_*$ [eV]

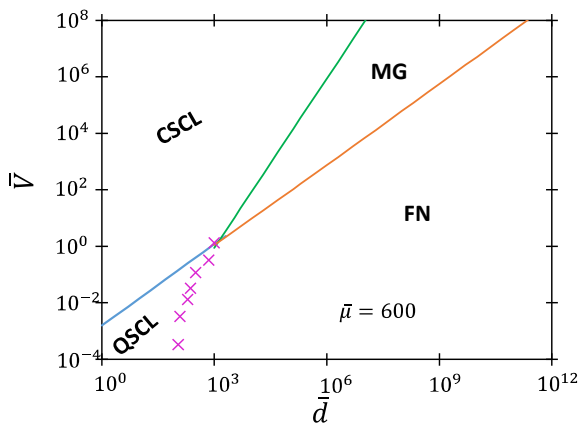


Figure 1. State diagram describing transitions between regimes.

## Results

Figure 1 demonstrates a state diagram showing transitions between the different regimes discussed here. The nexus occurring when these four regimes meet demonstrates a point of high instability when small perturbations in the system can shift the dominant emission behavior into any of the regimes. Using a thumbrule relating pressure to mobility for nitrogen by [11]  $v_d = \mu E$ ,  $v_e = 3.3 \times 10^6 \sqrt{E/p}$  gives the nexus in Figure 1 occurring at a pressure of  $\sim 1780$  Torr. Further experimental work around this nexus point is needed to completely understand the implications of this point and to determine better relationships between mobility and pressure.

## Conclusions

This work described a universal (gas independent) emission model predicting electron emission and gas breakdown from nanoscale up to the traditional PL, where one material-dependent parameter remains. Preliminary results identifying transitions between various emission mechanisms indicate a nexus point where the electron emission behavior can be pushed into any of the emission regimes with a slight perturbation in system conditions. Future work aims to assess the relevance and full implications of this nexus point to aid in device design and experimental setup.

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