Stability Analysis of VE Amplifiers Based on Determinant Equations

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Abstract: *A general method for calculating self-excitation thresholds for a large class of standing and traveling wave structures used in klystrons, traveling wave tubes, and other vacuum-electronic devices is presented. We determine circuit parameters of RF structures that are changed due to the presence of electron beam, and analyze stability of the obtained matrices. Determinant equations defined by such stability matrices are derived. The use of the method for structures with round and arbitrary geometry beam tunnels is discussed. Analytical stability evaluation described here greatly complements the large signal beam-wave interaction CHRISTINE and TESLA family codes as well as EM PIC codes such as NEPTUNE.*

Keywords: electron beam; circuit parameter matrix; multi-port network; rf structure; stability

Introduction

Vacuum electronic (VE) RF structures with interaction gaps and electron (e-) beams used for amplifying RF power can turn to unwanted self-excited oscillations. Conditions triggering oscillations depend on the structure geometry, gaps and e-beam parameters, and port terminations. An approach for modeling stability of VE devices using the structure gaps' impedance matrices for computing the Nyquist stability criterion has been reported [1].

This paper presents a method for calculating stability thresholds for the beam current and frequencies of self-oscillations, from the solutions of determinant equations that we obtain for the whole structure circuit parameters (*Z*- and *Y*-matrices) and the beam admittance matrix. The analysis also points to the part of the structure (interaction gaps, coupling slots, or power ports) causing the instability.

The impedance (*Z*-) and admittance (*Y*-) matrices, can be obtained with the finite element 3D electromagnetic code Analyst (National Instruments) and using the Z-matrix joining method for large complex structures [2]. The beam *Y*-matrix can be computed with the CHRISTINE code.

RF structure with electron beam

Z-matrix of an RF structure with interaction gaps describes RF voltages and currents produced at the gaps (lumped ports) and wave ports (input and output ports) in response to the one by one excitation of all ports of such a multiport network. One can analyze the Z-matrix (Z_1) to determine the structure resonant frequencies and oscillation and stability conditions.

E-beam traversing the interaction gaps induces RF voltages and currents at the gaps, which is described by the beam admittance matrix Y_2, defined to be circuit independent. This condition assumes no reflection of the beam-induced EM-wave in the gaps from the circuit. Because of the electrons directed motion through the structure, the Y_2-matrix is essentially triangular and invertible, with the beam impedance matrix given by $Z_2=Y_2^{-1}$.

E-beam injected in the RF structure alters the structure Z-parameters. We need to find the Z-matrix (Z_C) of the structure in the presence of the beam. One can then analyze the Z_C-matrix to determine the 'hot' structure resonant frequencies and stability (zero-drive self-excitation) conditions that differ from the 'cold' structure Z_1. Structures without e-beam (cold, passive) may have resonant frequencies but no self-excitation. The structure with e-beam can be treated as resulted from parallel connection of two multiport networks, merged at the gaps, and is designated in shorthand notation by $Z_1 \overline{\oplus} Z_2 \longrightarrow Z_C$.

The roots of the stability matrix determinant equations

$$\det Z_C = 0 \quad \text{and} \quad \det Y_C = 0 \quad , \qquad (1)$$

where Y_C is the combined (merged) structure admittance matrix, give the self-excitation conditions, caused by the current- and voltage-like resonances, respectively. From these roots one can determine the e-beam current stability thresholds along with the self-oscillation frequencies.

Circuit parameters of the merged structure

Assume that Z-matrices of two structures to be merged at a set of common lumped ports (gaps) are given as Z_1 and Z_2 and that of the combined structure as Z_C. The network diagram in Fig. 1 shows the port indexing order and categories. Three categories of ports are defined as essential, shared, and merged. Essential ports belong to Z_1 and remain present in Z_C after merging. Shared ports belong to Z_2 and remain present in Z_C after merging. Merged ports are those to be joined and are therefore present in equal numbers in both Z_1 and Z_2, and remain present merged in Z_C after joining. Essential and shared ports can be wave or lumped ports. Merged ports at the interaction gaps are always lumped ports.

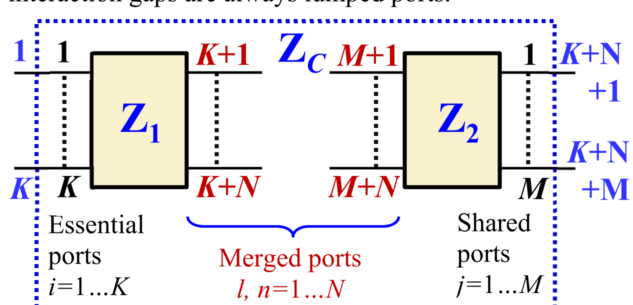

Fig. 1. Block diagram of parallel connection network showing port indexing and categories.

Partition the Z-matrices into block forms grouped according to the introduced port categories:

$$Z_1 = \begin{pmatrix} Z_{1_{EE}} Z_{1_{EJ}} \\ Z_{1_{JE}} Z_{1_{JJ}} \end{pmatrix} \quad Z_2 = \begin{pmatrix} Z_{2_{SS}} Z_{2_{SJ}} \\ Z_{2_{JS}} Z_{2_{JJ}} \end{pmatrix} \quad Z_C = \begin{pmatrix} Z_{C_{EE}} Z_{C_{ES}} Z_{C_{EJ}} \\ Z_{C_{SE}} Z_{C_{SS}} Z_{C_{SJ}} \\ Z_{C_{JE}} Z_{C_{JS}} Z_{C_{JJ}} \end{pmatrix}$$

where subscripts *E, J,* and *S* denote the essential, merged, and shared ports. Applying Kirchhoff's laws together with the continuity conditions, eliminate the voltage and current variables associated with the merging ports, and obtain

$$Z_C = \tag{2}$$

$$\begin{pmatrix} Z_{1_{EE}} - Z_{1_{EJ}} Y_+ Z_{1_{JE}} & Z_{1_{EJ}} Y_+ Z_{2_{JS}} & Z_{1_{EJ}} Y_+ Z_{2_{JJ}} \\ Z_{2_{SJ}} Y_+ Z_{1_{JE}} & Z_{2_{SS}} - Z_{2_{SJ}} Y_+ Z_{2_{JS}} & Z_{2_{SJ}} Y_+ Z_{1_{JJ}} \\ Z_{2_{JJ}} Y_+ Z_{1_{JE}} & Z_{1_{JJ}} Y_+ Z_{2_{JS}} & Z_{1_{JJ}} Y_+ Z_{2_{JJ}} \end{pmatrix}$$

where $\quad Y_+ = Z_+^{-1}$ and $Z_+ \equiv Z_{1_{JJ}} + Z_{2_{JJ}}$.

For the combined admittance matrix resulted from merging $Y_1 \oplus Y_2 \longrightarrow Y_C$, where $Y_1 = Z_1^{-1}$, Y_C is found to be

$$Y_C = \begin{pmatrix} Y_{1_{EE}} & 0 & Y_{1_{EJ}} \\ 0 & Y_{2_{SS}} & Y_{2_{SJ}} \\ Y_{1_{JE}} & Y_{2_{JS}} & Y_{1_{JJ}} + Y_{2_{JJ}} \end{pmatrix} \tag{3}$$

In the case of e-beam, there are no shared ports in the Z_2 and Y_2 matrices and therefore Z_C (2) and Y_C (3) reduce to

$$Z_C = \begin{pmatrix} Z_{1_{EE}} - Z_{1_{EJ}} Y_+ Z_{1_{JE}} & Z_{1_{EJ}} Y_+ Z_{2_{JJ}} \\ Z_{2_{JJ}} Y_+ Z_{1_{JE}} & Z_{1_{JJ}} Y_+ Z_{2_{JJ}} \end{pmatrix} \tag{4}$$

$$Y_C = \begin{pmatrix} Y_{1_{EE}} + Y_L & Y_{1_{EJ}} \\ Y_{1_{JE}} & Y_{1_{JJ}} + Y_{2_{JJ}} \end{pmatrix} \tag{5}$$

where Y_L is the wave ports' load admittance matrix. The combined Z_C-matrix (4) with the account of the wave ports termination Y_L is given by the transformation, (7) in [2].

Self-excitation equations

From the solutions for the voltage and current at the ports obtained for the block-partitioned matrices, arrive at the self-excitation conditions for Y_C and Z_C given by (5) and (4), respectively for voltage- (selected equations shown)

$$\left[\begin{array}{ll} \det\left(Y_{1_{JJ}} + Y_{2_{JJ}}\right) = 0 & (6.1) \\ \det\left(Y_{1_{EE}} + Y_L - Y_{1_{EJ}}\left(Y_{1_{JJ}} + Y_{2_{JJ}}\right)^{-1} Y_{1_{JE}}\right) = 0 & (6.2) \end{array}\right.$$

and for current-like resonances

$$\left[\begin{array}{ll} 1/\det\left(Z_{1_{JJ}}^{-1} + Y_{2_{JJ}}\right) = 0 & (7.1) \\ \det\left(Z_{C_{EE}} - Z_{C_{EJ}}\left(Z_{1_{JJ}}^{-1} + Y_{2_{JJ}}\right) Z_{C_{JE}}\right) = 0 & (7.2) \\ 1/\det(\mathbb{I} + Y_L Z_C) = 0 & (7.3) \end{array}\right.$$

where $\mathbb{I}$ is the identity matrix. In such representation, (6.1) and (7.1) describe local self-excitations due to the gaps, (6.2) and (7.2) consider the rest of the structure, and (7.3) infers the wave port terminations. A complete set of the self-excitation equations includes all possible partitions of the Y_C, Z_C-matrices formed by adding to JJ-block essential ports from EE-block with all possible permutations as well as the reciprocal determinants.

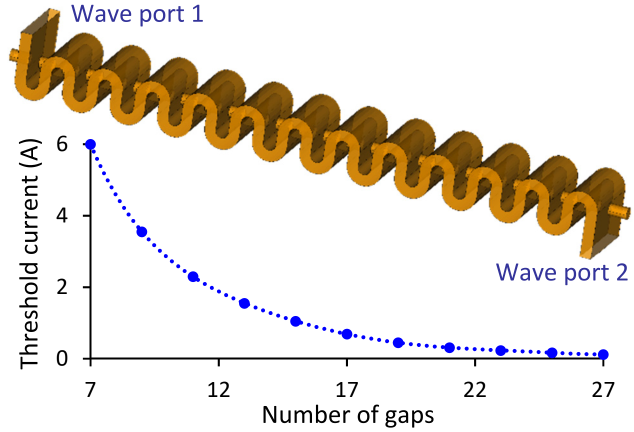

Fig. 2. An example multi-gap structure (top) and the obtained stability conditions (bottom).

The beam current self-excitation thresholds found using (6) and (7) for the example multigap K_a-band serpentine waveguide with a 0.32-mm radius beam in a 0.4-mm radius beam tunnel are shown in Fig. 2. The calculations assume the 23.5 kV beam voltage, at which, with respect to the structure frequency dispersion properties, the backward-wave self-oscillation instability may be triggered. For each model with the varying number of gaps, we compute the required structure Z_1 and beam Y_2-matrix that are frequency dependent. The latter also depends on the beam current.

Beam tunnel of arbitrary geometry

In case of a round beam tunnel CHRISTINE can compute the required beam admittance Y_2-matrix. For the arbitrary geometry beam tunnel, we suggest first to compute the circuit parameters for an auxiliary structure constructed from the beam tunnel and gaps only, without and with the e-beam. The beam-loaded circuit parameters are obtained then from the short-time PIC simulation with NEPTUNE. From these data we derive the Y_2-matrix for the beam tunnel of arbitrary geometry, needed as input for defining the self-excitation equations.

The stability threshold calculations, multibeam structures, and PIC simulations will be shown at the Conference.

Acknowledgements

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References

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