Adjoint Approach to Optimization and Sensitivity Analysis of Beam Wave Interaction in Vacuum Electronic Devices

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Abstract: *We demonstrate a new approach to optimization and sensitivity analysis of beam-wave interaction in any vacuum electronic device (VED) that consists of a circuit interacting with a linear beam through a series of gaps. The basis of the method is a consequence of the Hamiltonian form of the equations that govern the beam-wave interaction, which implies the conservation of symplectic area for two perturbed solutions. Using this property of the system we have derived a relationship between the perturbed solution and an adjoint solution to the linearized equations. We show that proper selection of the adjoint solution allows obtaining compact symplectic equations. Consequently when the adjoint solution is obtained using a simulation code, it may be used to evaluate the multi-dimensional derivatives needed for efficient optimization and for sensitivity analysis. For sensitivity analysis the single adjoint solution allows to construct a sensitivity function what acts like a type of Green's function for many variable parameters of the system. The approach applies to small or large signal operation of standing and traveling wave devices using either single or multiple round beams.*

Keywords: adjoint solution; impedance matrix; tolerance; vacuum electron device.

Introduction

Vacuum electronic amplifier technology is constantly pushed to higher operating frequencies, broader bandwidths, increased power and efficiency levels, and higher specific power (ratio of output power to total weight of device). To meet these requirements, new vacuum electronic devices with advanced performance characteristics must be explored and existing device designs must be optimized to reach the best possible performance. Further, the devices have become more and more complex with many factors affecting device performance and stability. As a consequence, an optimal design must be found in a large, high-dimensional parameter space. As manufacturing tolerances also play an increasingly significant role as the operating frequencies of VEDs increase into the millimeter-wave regime, successful optimizations must include a means to evaluate a given design's sensitivity to unavoidable fabrication variations. Modern VED design is based on numerical simulations of the particular device's performance. Through last decade several highly efficient design computer codes have been developed by the NRL/Leidos team, including the most recent codes TESLA-Z and CHRISTINE-Z [1] suitable for modeling and design of wide class of vacuum electronic amplifiers. To achieve higher performance and efficiencies of amplifiers, the computer based design process should also include multi-parameters optimization enabling trade-offs between subsystem capabilities. Current optimization packages require a large number of computationally-intensive evaluations of a configuration's performance in order to find the optimum parameters in a multidimensional parametric space. In this work we extend the adjoint approach to particle-field dynamic follow early reported in [2,3] in order to provide fast calculations of multi-dimensional derivatives needed for more advanced gradient based optimization algorithms as well as for evaluation of sensitivity functions.

Adjoint Relation for Beam-Wave Interaction

We consider an electron beam interacting electromagnetically with a circuit through the electric fields produced by circuit voltages appearing across gaps separated by drift spaces as schematically illustrated in Fig. 1. We assume that a steady state solution exists, with all fields oscillating at frequency ω. It is more transparent to illustrate the idea of the adjoint approach using self-consistent 1D model using in CHRISTINE-Z code. If we denote the voltages across the n-th gap by V_n then the energy $E_k \equiv mc^2\gamma_k$ of the k-th particle in the beam will satisfy

$$\frac{dE_k}{dz} = q\sum_n V_n e_n(z-z_n)\, e^{-i\omega t_k(z)} - iqE_0 e^{-i\omega t_k(z)} \langle e^{-i\omega t_{k'}(z)}\rangle_{k'} + c.c. \tag{1}$$

relationship where $q=-e$ is the electron charge, $e_n(z)$ is a normalized shape function for the gap field, with units of inverse length (See [1]), z_n is the axial location of the center of gap n, $t_k(z)$ is the arrival time of particle k at location z, and $E_0 = mv_b\omega_q^2/q\omega$, where v_b is the beam velocity and ω_q is the reduced plasma frequency of the beam. The notation $\langle\ldots\rangle_k$ denotes an average over all particles in the beam. The E_0 term represents the AC space charge field.

The arrival time $t_k(z)$ satisfies

$$\frac{dt_k}{dz} = \frac{1}{\mathrm{v}(E_k)} \tag{2}$$

where $v(E)$ is the axial particle velocity.

We assume that the gap voltages may be obtained from the currents induced by the beam at each gap and an input signal at a port through a linear circuit equation of the form

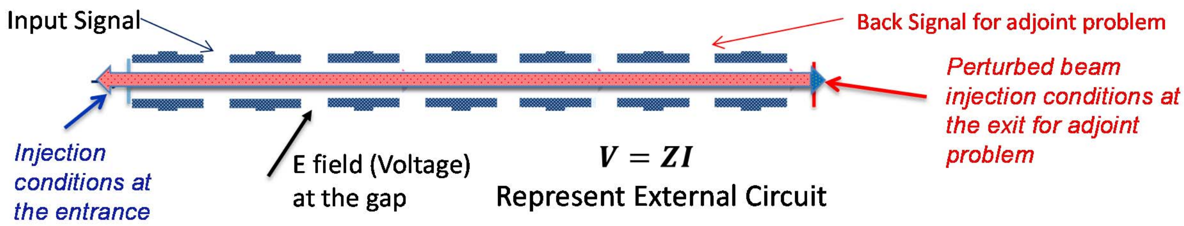

Figure 1. Schematic view of the beam-wave interaction area (beam tunnel with gaps representing external circuit) where direct and adjoint solutions are constructed.

$$\boldsymbol{V} = \boldsymbol{Z}\boldsymbol{I} \tag{3}$$

where $\boldsymbol{V}$ is a vector of voltages, $\boldsymbol{I}$ is a vector of currents, and $\boldsymbol{Z}$ is an impedance matrix, which is assumed known. $\boldsymbol{V}$ and $\boldsymbol{I}$ include both gap and port currents.

The Eqns. (1) and (2) can be expressed in Hamiltonian form:

$$\frac{dE_k}{dz} = -\frac{\partial}{\partial t_k} P(E_k, t_k; z), \qquad \frac{dt_k}{dz} = \frac{\partial}{\partial E_k} P(E_k, t_k; z) \tag{4}$$

where the Hamiltonian is the axial canonical momentum, P in this case. Using the symplectic area conservation law we derived the relationship between base solution, adjoint solution and perturbed solution:

$$2\sum_{ports}[\delta I_n^{A*}\delta V_n - c.c.] = \frac{\omega}{iq}\frac{I}{N}\sum_i(\delta t_i^A \delta E_i - \delta t_i \delta E_i^A)_{z=0} + \left(\sum_{gaps}[\delta I_n^{A*}\delta V_n + \delta I_n \delta V_n^{A*}] + 2\sum_{ports}\delta I_n^{A*}\delta V_n - c.c.\right) + I\int_0^L dz[(\delta\psi^{A*}V_n + \psi^*\delta V_n^{A*})\delta e_n(z) - c.c.] \tag{5}$$

where ψ and $\delta\psi$ ω are the bunching factor for base solution and its perturbation for adjoint solution. Superscript A denotes adjoint quantities. Note, that compact form of Eqn. (5) is obtained if we require the adjoint voltages and currents to satisfy

$$\delta \boldsymbol{V}^A = -\boldsymbol{Z}^\dagger \delta \boldsymbol{I}^A \tag{6}$$

where † denotes the adjoint (conjugate transpose). If we additionally require that the adjoint solution have the property that $\delta I_{input}^A = 0$, then we may compute the full adjoint solution using CHRISTINE-Z by (1) applying a small signal at the output port ($\delta I_{output}^A \neq 0$) and (2) integrating the equations of motion backwards from the output end to the input end, and computing the resulting gap voltages and currents. The initial conditions at $z = L$ for the particles for the backwards integration are the same as the final conditions in the unperturbed (base case) simulation.

The significance of adjoint relationship (5) is that we do not need to solve many times the equations (1)-(3) to find the effect of small perturbations of various beam-wave interaction parameters. We solve once the base case and once the adjoint case and find the real perturbed solution by multiplication of Eqn. (5) to distribution function of parameter variations and integration.

Numerical Properties and Applications of the Adjoint Relation

Study on numerical properties of the adjoint solution calculated by CHRISTINE-Z code has been done for several cases of traveling wave amplifiers with moderate and high gain. It was shown that perturbations in bunching factor and voltages and currents on gaps for adjoint case are linear with respect to amplitude on back injected signal whiting wide range of amplitude variations.

We will present at the conference examples of evaluation of sensitivity functions for cases when direct approach is impractical due to large number of varying parameters. One example is the effect of velocity spreads in electron beam. Sensitivity function of amplifier performance with respect to velocity spreads can be calculated from Eqn. (5) using first term in the right hand side of the equation by multiplying to velocity distribution function. The adjoint approach allows to find the effect of spread using one base and one adjoint solutions. In contrast, any direct approach will require multiple runs with many particles included in simulations to address spreads accurately. Second example is evaluation of sensitivity function with respect to variations of the shape of gap fields *δe(z)* to study the effect of variation in shape of resonators' noses.

Acknowledgements

This work supported by the U.S Office of Naval Research. Distr. A: Approved for public release, distribution is unlimited.

References

1. I.A. Chernyavskiy, T.M. Antonsen, Jr., J.C. Rodgers, A.N. Vlasov, D. Chernin and B. Levush., "Modeliing Vacuum Electronic Devices Using Generalized Impedance Matrices", IEEE TED, Vol. 64, No. 2, pp. 536-542, Feb. 2017.
2. T.M. Antonsen, Jr., D. Chernin, and J. Petillo, "Adjoint approach to beam optics sensitivity based on Hamiltonian particle dynamics," Physics of Plasmas 26, 013109, 2019.
3. D. Chernin, T.M. Antonsen, Jr., "Efficient Evaluation of the Effects of Manufacturing Tolerances on VED Performance," Proc. IVEC 2018, Monterey, CA.