An Efficient Eigensolver for Extended Interaction Klystrons Based on Finite Element Method

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Abstract: *This paper presents an efficient eigensolver based on finite element method (FEM) for metallic-lossy multigap cavities in extended interaction klystrons (EIKs). By modeling a W-band sheet-beam EIK (SBEIK), the eigensolver is validated. Moreover, it is found that our eigensolver is much more efficient than the widely used commercial FEM code i.e. HFSS, which would be very useful for the design of multigap cavities of EIKs.*

Keywords: characteristic impedance, eigenanalysis, extended interaction klystrons, multigap cavity, finite element method

INTRODUCTION

The multigap cavity is one of most important parts of the extended interaction klystron (EIK), because it determines the power, bandwidth and efficiency of the EIK. A critically important step in the design of EIK is conducting eigenanlysis for the multigap cavity using computational electromagnetic methods such as finite element method (FEM) [1]. However, the eigenanlysis of lossy multigap cavity by FEM often places a heavy burden on the time consumption owing to the following two reasons. First, in the eigenanalysis of multigap cavity, a large number of spurious dc modes corresponding to the irrotational solutions of the vector wave equation occur at zero frequency. The spurious dc modes make it difficult to calculate the smallest eigenvalues and even result in failing to converge to the desired eigenmodes. Second, because of the metallic lossy of multigap cavity, the eigenanalysis of the multigap cavity has to solve a large scale *non-linear* generalized eigenvalue problem. To overcome defects mentioned above, we present a new 3-D eigensolver, which adopts a hierarchical vector FEM with second-order basis, for the metallic-lossy multigap cavity. The application of this eigensolver not only obtains the cold parameters accurately, but also reduces the CPU time and memory requirement.

THEORY AND FORMULATION

The boundary value problem (BVP) for the finite-element analysis of metallic-lossy cavities of EIK can be written as

$$\nabla\times\mu_r^{-1}\nabla\times\vec{E}-k_0^2\varepsilon_r\vec{E}=0,\quad \text{in}\ \ \Omega \tag{1}$$

$$\vec{n}\times\mu_r^{-1}\nabla\times\vec{E}=\left(jk_0Z_0/Z_s\right)\vec{n}\times\left(\vec{E}\times\vec{n}\right),\quad \text{on}\ \ \Gamma_{\text{SIBC}} \tag{2}$$

$$\nabla\cdot\left(\varepsilon_r\vec{E}\right)=0,\quad \text{in}\ \ \Omega \tag{3}$$

where k_0 is the free-space wavenumber, $\vec{n}$ is the outgoing normal on the boundary surfaces, and the metal surface impedance $Z_s=(1+j)\sqrt{(\omega\mu_0)/(2\sigma_m)}$. Equation (2) is the imposed surface impedance boundary condition (SIBC). If we perform a divergence operation for the vector wave equation and set $k_0=0$, the divergence-free constraint equation (3) may no longer hold, which leads to the spurious dc modes appear. Hence, it is necessary to impose (3) on (1) and (2) as a constraint equation to eliminate the dc modes.

By using Galerkin's procedure [1], the weak formulation for BVP (1)-(3) is first obtained, i.e.

$$\begin{aligned}&\iiint_\Omega(\nabla\times\vec{v}\cdot\mu_r^{-1}\nabla\times\vec{E})dv-k_0^2\iiint_\Omega(\vec{v}\cdot\varepsilon_r\vec{E})dv\\&-(jk_0Z_0/Z_s)\iint_{\Gamma_{\text{SIBC}}}\vec{n}\times(\vec{v}\times\vec{n})\cdot\vec{n}\times(\vec{E}\times\vec{n})ds=0\end{aligned} \tag{4}$$

$$\iiint_\Omega(\nabla\phi\cdot\varepsilon_r\vec{E})dv=0 \tag{5}$$

where $\vec{v}$ and ϕ are test basis functions, and $\vec{v}\in\mathbf{V}$, and $\nabla\phi\in\mathbf{G}\subset\mathbf{V}$ (**V** is the basis functions space). If basis space **V** can be completely decomposed into a pure gradient part **G** and a rotational part **R**, and $\mathbf{V}=\mathbf{G}\cup\mathbf{R}$ and $\mathbf{G}\cap\mathbf{R}=\varnothing$, the spurious dc modes can be very efficiently removed without directly solving equation (5). The decomposition of basis functions space **V** is realized by adopting the hierarchical vector basis function proposed by Sun et al. [2], and implementing the tree-cotree splitting [3] on all metallic-lossy boundaries of multigap cavity. The detail of this procedure will be given in the presentation. Then, (4) can be now expressed in matrix form as:

$$\mathbf{S}x-k_0^2\mathbf{T}x-j\sqrt{k_0}\mathbf{L}x=0 \tag{6}$$

where $\mathbf{S}_{ij}=\iiint_\Omega(\nabla\times\boldsymbol{N}_i\cdot\mu_r^{-1}\nabla\times\boldsymbol{N}_j)dv$,

$\mathbf{T}_{ij}=\iiint_\Omega(\boldsymbol{N}_i\cdot\varepsilon_r\boldsymbol{N}_j)dv$,

$\mathbf{L}_{ij}=\sqrt{2\sigma_mZ_0}\,/\,(1+j)\iint_{\Gamma_{\text{SIBC}}}(\vec{n}\times\boldsymbol{N}_i\times\vec{n})\cdot(\vec{n}\times\vec{n}\times\boldsymbol{N}_j)ds$,

and $\boldsymbol{N}_i$ represents the *i*th basis function. It is clear that (6) is a fourth-order nonlinear generalized eigenvalue problem. We use a symmetric linearization technique to transform (6) into a symmetric linear generalized eigenvalue problem as:

$$\begin{bmatrix}\mathbf{0}&\mathbf{0}&\mathbf{T}&\mathbf{0}\\\mathbf{0}&\mathbf{T}&\mathbf{0}&\mathbf{0}\\\mathbf{T}&\mathbf{0}&\mathbf{0}&\mathbf{0}\\\mathbf{0}&\mathbf{0}&\mathbf{0}&\mathbf{S}\end{bmatrix}\begin{bmatrix}\lambda^3x\\\lambda^2x\\\lambda x\\x\end{bmatrix}=\lambda\begin{bmatrix}\mathbf{0}&\mathbf{0}&\mathbf{0}&\mathbf{T}\\\mathbf{0}&\mathbf{0}&\mathbf{T}&\mathbf{0}\\\mathbf{0}&\mathbf{T}&\mathbf{0}&\mathbf{0}\\\mathbf{T}&\mathbf{0}&\mathbf{0}&j\mathbf{L}\end{bmatrix}\begin{bmatrix}\lambda^3x\\\lambda^2x\\\lambda x\\x\end{bmatrix} \tag{7}$$

where $\lambda=\sqrt{k_0}$. From (7), it can be seen that the matrix dimension of linear generalized eigenvalue problem is *four times* than that of (6). Hence, solving (7) is very inefficient. To address this difficulty, in the solution procedure of (7) using traditional implicitly restarted Arnoldi method, we apply a new p-type multigrid preconditioner and an inverse-based multifrontal block incomplete LU factorization. After

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removing the spurious dc modes, the solution of (7) can be further improved. Consequently, a remarkable efficient eigensolver for metallic-lossy multigap cavity of EIK is obtained. By using this eigensolver, arbitrary resonant mode and its characteristic impedance R/Q in the cavity are able to be calculated very accurately and fast.

Numerical Results

To validate the accuracy and the efficiency of the proposed eigensolver, we compare the results and computational performance with HFSS (AnsysEM 19.2) by simulating the multigap cavity of a W-band sheet-beam EIK (SBEIK) [4]. All simulations are conducted on a win10 64-bit intel core 2 2.3-GHz and 16-GB RAM laptop, meanwhile second-order basis function has been selected in HFSS.

The structure of the multigap cavity is shown in Fig. 1, and the model is consisted of the vacuum and the copper. The distance P between the adjacent gaps is designed to be 1.5 mm, and the gap width d is designed to be 0.38 mm. Fig. 2 shows the first 14 modes error analysis of the resonant frequency of the W-band SBEIK computed by two simulators. Then, the performance comparison is shown in Table I, from which we can see that the proposed eigensolver achieved a speedup of up to quadruple compared by HFSS.

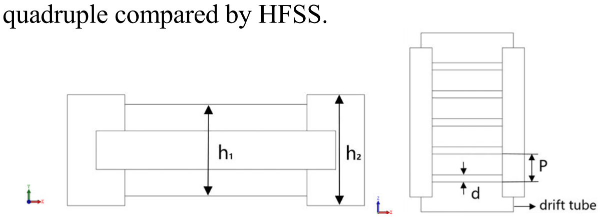


Fig. 1. Structure and CAD model of the W-band SBEIK.

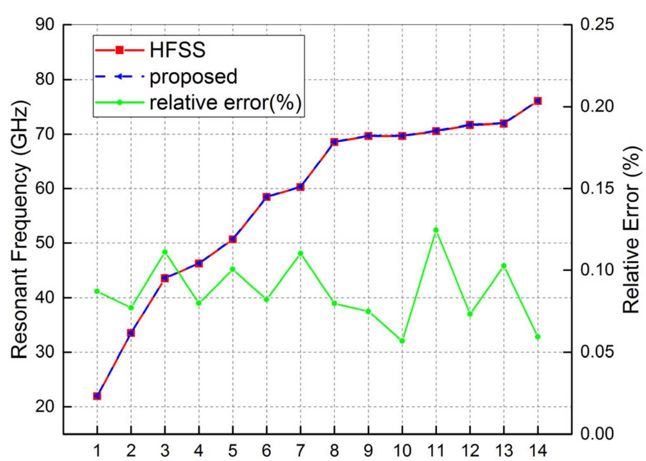


Fig. 2. Comparison of the resonant frequency computed by two simulators.

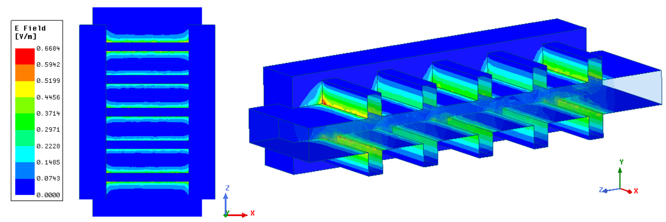
TABLE I. PEFORMANCE COMPARISON

Simulator	Degrees of freedom	Matrix Size	CPU time (hh:mm:ss)
Proposed	112,332	996,190	00:12:17
HFSS	111,423	965,661	00:51:29

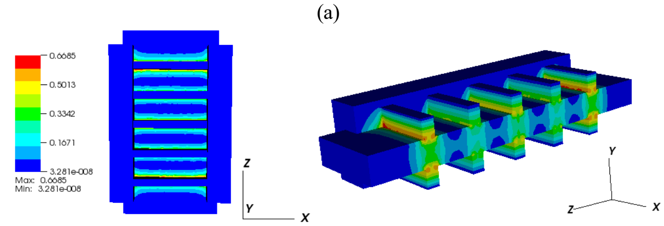
Finally, we compare the parameters of the W-band SBEIK working in 95GHz. The E-field distribution of the model calculated by two solvers is plotted as shown in Fig. 3. The integral line is selected on the upper surface of gaps of the drift tube, and Table II shows the relative errors in converged mesh.

Conclusion

This paper has presented an efficient eigensolver for simulation of the multigap cavity of EIKs. Then a W-band SBEIK model is simulated to demonstrate the accuracy of the proposed eigensolver. The application of our eigensolver dramatically reduces the required computing time compared with commercial simulator HFSS, which would be very useful for the design of high-performance EIKs.



(a)



(b)

Fig. 3. Displacement distribution of E-field compared with HFSS. Distribution of E-field simulated by HFSS (a). Distribution of E-field simulated by proposed eigensolver (b).

TABLE II. PROPERTIES OF THE CAVITY OF A W-BAND SBEIK

Simulators / Quantities	HFSS	Proposed	Relative error(%)
Frequency(GHz)	95.0932	95.1225	0.03
Q	1,303.48	1,313.11	0.74
R/Q(Gap 1)	4.2479	4.2298	0.43
R/Q(Gap 2)	1.7782	1.7960	1.00
R/Q(Gap 3)	2.9559	2.9723	0.55
R/Q(Gap 4)	1.7730	1.7751	0.12
R/Q(Gap 5)	4.1958	4.1715	0.58

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