Stopband and Coupling-Coefficient Estimation for Asymmetries in Helical Delay-Lines

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Abstract

The π-point stopband in helical delay-lines due to asymmetries is analyzed. For this, we model the structure as a conductor-backed coplanar waveguide. The stopband is evaluated from the effective transmission-line parameters and the results are compared to full-wave simulation. Although the approach is quite general, we only consider non-ideal support-rod positions in this contribution. Finally, the stopband is modeled by applying the coupled-mode theory, including a simple coupling-coefficient estimation.

INTRODUCTION

Asymmetries in the beam-wave interaction region of helix traveling-wave tubes (TWTs) lead to a coupling of the forward- and backward-propagating modes. Asymmetries are for instance caused by off-centered electron beams or by non-ideal delay-lines. In case of beam-wave synchronism at the so-called π-point, where the phase advance in the ω-β diagram is π, parasitic π-mode oscillations (PMOs) are very likely to be excited [1] as any asymmetry lowers the oscillation threshold significantly compared to that of backward-wave oscillations. Although inherent beam-wave synchronism at the π-point can be avoided during the tube design, providing a stable tube in the case of small-signal drive levels in general, high beam-efficiency operation alters the beam-wave synchronism and the interaction can shift into the π-mode region. The resulting instability is known as drive-induced oscillation (DIO).

In this contribution, we propose computationally efficient modeling and evaluation of the stopband caused by misaligned support rods in helical delay-lines as a starting point of an in-depth numerical analysis of PMO and DIO. First, similarly to [2] and [3], the unit winding is modeled by transmission lines (TLs) where the corresponding parameters are in our case gained from an analogy to coplanar waveguides (CPWs). The resulting dispersion is compared to full-wave simulation. Subsequently, the stopband is expressed by a coupling coefficient between the forward- and backward wave like in [1]. This allows to accurately model the stopband by means of the coupled-mode theory from [4].

EQUIVALENT-CIRCUIT MODEL

At typical TWT operating frequencies, the electromagnetic helix-field is spatially distributed across the barrel's volume. At higher frequencies, i.e., around the π-point frequency f_π here, it is more concentrated between the windings and has

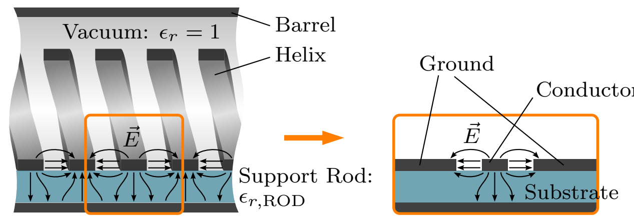

Figure 1. Helical delay-line and equivalent conductor-backed CPW with electric field distribution at f_π.

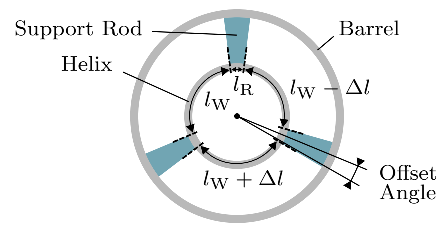

Figure 2. TL lengths from the delay-line geometry.

a phase advance of π per helix winding. Thus, the helix behaves much like a conductor-backed CPW for even mode propagation as illustrated in Figure 1. From the cross section of the unit winding shown in Figure 2, one can then easily derive the equivalent TL model in Figure 3. It consists of six TLs representing the three support-rod sections and the segments in between. Besides the misalignment of a single support rod, causing a line-length imbalance of Δl, the model also includes other asymmetries, such as a variation of the support rod permittivity or size. In case of a vane loading, additional TLs are needed.

The CPW geometry can directly be determined from the helical delay-line, the conductor cross-section being that of the helix wire and the substrate height that of the support-rod. The relative permittivity of the CPW substrate is either $\epsilon_{r,\mathrm{ROD}}$ or one. The characteristic impedance and the effective relative permittivity of the CPW can be computed from [5]. The line lengths have to be chosen according to an effective helix radius which ranges somewhere between the inner and the average helix-radius at the π-point. The choice of the effective helix radius possibly shifts f_π by a few percent but hardly affects the stopband behavior. The uncertainty obviously increases with the helix-wire thickness. Here, for better comparison, the value is adjusted to fit f_π obtained from full-wave simulation.

Replacing the TLs by lumped elements from [6] and concatenating their chain matrices yields the transmission matrix of

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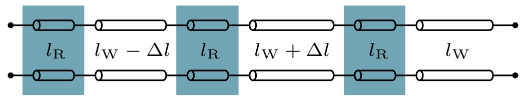

Figure 3. Transmission-line model.

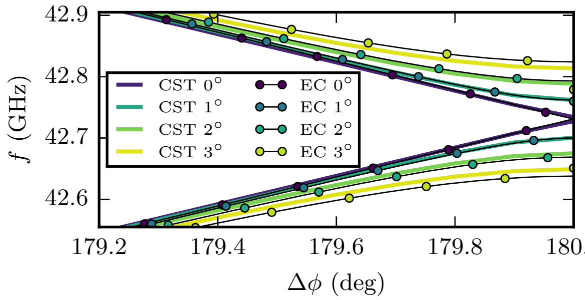

Figure 4. π-point stopband from simulation and from the equivalent circuit (EC) for different support-rod offset-angles.

the equivalent circuit model. Subsequently, the stopband is obtained by computing its eigenvalues. As Figure 4 shows, the results compare well with those from an eigenmode simulation in CST [7] for different support-rod offsets.

COUPLING COEFFICIENT ESTIMATION

In the preceding section, the π-point stopband was determined from the equivalent circuit model. In the next step, it is modeled by means of the coupled-mode theory from [4] and directly linked to a frequency-independent coupling-coefficient c between the propagating modes of the symmetric helix

$$\frac{\partial}{\partial z}\begin{pmatrix} a_1' \\ a_2' \end{pmatrix} = \begin{bmatrix} -j\beta_1 & c \\ c^* & -j\beta_2 \end{bmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \qquad (1)$$

where a_1 and a_2 are the complex amplitudes of the forward and the backward wave, respectively, and β_1 and β_2 their respective propagation constants. The prime denotes coupled waves. The propagation constants of the coupled waves follow from the eigenvalues of the matrix in Equation (1) with

$$\beta_{1,2}' = \frac{\beta_1 + \beta_2}{2} \pm \frac{1}{2}\sqrt{(\beta_1 - \beta_2)^2 - 4\,|c|^2}. \qquad (2)$$

By approximating the slope of the dispersion curve by f_π/π and equating the square-root term in Equation (2) to zero, the coupling coefficient can be calculated from the relative bandwidth $\Delta f / f_\pi$ of the stopband to

$$|c| \approx \frac{\Delta f}{f_\pi} \cdot \frac{\pi}{2p}. \qquad (3)$$

Here, p denotes the helix pitch. Figure 5 compares the dispersion obtained from the equivalent circuit and from the coupled-mode theory. The decoupled propagation constants β_1 and β_2 are extracted from the equivalent circuit for the symmetric helix and the applied coupling coefficient is gained from the equivalent circuit with the respective support-rod misalignment. The curves are almost identical and thus, the coupled-mode theory and the equivalent-circuit

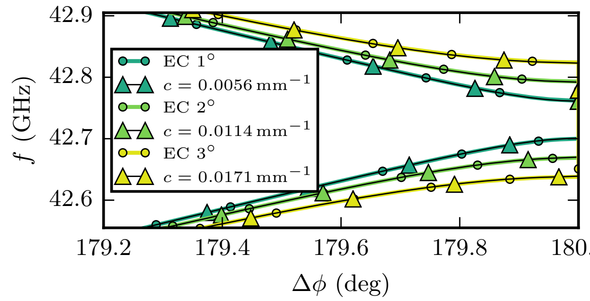

Figure 5. Stopband at the π-point from the equivalent circuit (EC) and from the coupled-mode theory.

model are suitable for a computationally efficient as well as simple stopband and coupling-coefficient estimation for the analysis of delay-line asymmetries.

CONCLUSION

The helical delay-line of TWTs is modeled as a conductor-backed coplanar waveguide around its π-point frequency. From the various delay-line asymmetries the model can account for, only support-rod misalignments are considered here. The results match well with full-wave simulation. Furthermore, the stopband is modeled by means of the coupled-mode theory, which allows to easily extract the coupling coefficient. Compared to geometry-driven approaches, this work yields an appropriate and computationally-efficient representation of the stopband, and is thus a useful supplement of fast TWT simulation tools.

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